PEDAGOGY OF MATHEMATICS

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Unit I CONCEPT, NEED, OBJECTIVES AND SCOPE

1.0 Introduction

Mathematics is the creation of human minds. Because mathematics is made by persons and exists only in the minds it must be made or remade in the mind of each person who learns it. In this mathematics can only be learnt by being created. Mathematics is not just about number and space. The traditional definitions, meaning and the modern thinking about the nature of mathematics such as the language of mathematics and symbolism, logical structure, precision and abstraction are considered in this unit as you, the student teacher, will have to be familiar with the definitions and the nature of mathematics to become a good and efficient teacher of mathematics.

1.1. MEANING AND DEFINITION OF MATHEMATICS

We all know that mathematics is defined in simple terms as the science of quantity, measurement and spatial relationships. It is a systematized, organized and exact branch of sciences. It deals with quantitative facts, relationships as well as with problems involving space and form. The dictionary meaning of mathematics is that it is either the science of number and space or the science of measurement, quantity and magnitude-also the logical study of shape, arrangement and quantity.

Benjamin Peirce expressed it more explicitly as "Mathematics is the science which draws necessary conclusions."According to Comte, Mathematics is the science of indirect measurement. Distance between planets, diameter of an atom and rate of plant growth are some of the indirect measurements made.

For Smith "Mathematics is the rock upon which art and science of the world rests". According to Gauss, "Mathematics is the queen of sciences and Arithmetic is the queen of mathematics. Bacon says "Mathematics is the gate way and key to all sciences" and Kant feels that "Mathematics is the indispensable instrument of all physical researches."

Mathematics can be viewed as one of the great humanities because it is a method of expressing, explaining and communicating man's total behavior: it reigns as a queen of all sciences in its clear, rigorous and logical structure and in doing so, it serves as an ideal arid goal for perfection of other sciences. Moreover, it is also considered as the servant of all sciences, meaning thereby, it provides service to all other sciences. At the time of difficulties or hurdles they face for exact explanation and precise direction. Then mathematics has its utilitarian aspect because of its numerous applications. It is an instrument to train the mind.

1.1 NATURE OF MATHEMATICS: ITS CHARACTERISTICS

From various definitions and descriptions about mathematics it is easy to know and understand the characteristics features of mathematics. They are: abstractions, language and symbolism, logical structure and precision. We shall discuss these characteristics in detail now.

1.1.1. Logical Sequence

Mathematics is much more than counting, computing, drawing figures, using symbols with mysterious code. It is a way of thinking, a way of reasoning. Sometimes mathematics involves experimentation and observation but most of the cases it concerns with deductive reasoning. Mathematics is not just adding of figures all day long, which can best be done by machines. It involves mainly logical reasoning.

By reasoning in mathematics it can be proved that if something is true then something else must be true. For example

A proposition as "If a quadrilateral is a square, then it is a parallelogram" can be written as for any quadrilateral x, if x is a square, then x is a parallelogram; which is true?

By logical reasoning it is possible to explore the various ways in which a problem can be tackled or solved. Sometimes one can show by reasoning that the problem has no solution.

For example

"Place 36 marbles in 9 boxes such that each box must contain odd number of marbles" has no solution.

1.1.2 Structure

The inter related concepts and nature are becoming the structure of mathematics. Earlier, mathematics was used to count and measure, then it becomes the science of structure. In counting, there is a sequence viz number theories, subsets numbers, counting numbers, Natural numbers, Whole numbers, Integers, Rational Numbers, Irrational Numbers, Real Numbers and Complex Numbers. This is called the structure of Mathematics.

When the numbers undergo in the fundamental operations by using +, -, X, $_4$ and some basic rules, it will get some principles and properties.

For e.g.: 3 4 = 4x3 [called reversible property]

These so many mathematical structures are found like Algebraic structures and topological structures in mathematics.

There is a closer relationship among the concepts and theorems in mathematics. In this structure, first elements are formulated and then operations and properties are created according to the elements.

In other words, structures are used to create a new branch in mathematic. For e.g.: Correlating the Algebra and Geometry, We get "Analytical Geometry".

1.1.3. Precision

Mathematics has its own precision, that is why its called an exact science. Mathematically obtained result are either accepted or rejected, either correct or incorrect there is no double stand. Even in the case of approximation, the result of mathematics can be had to any degree of accuracy required, Measures at mathematics are precise to within certain specified units as 1/10 of 1cm, 500 Rs, 50 paise etc. Numbers are precise to units, tenths, hundredths etc. A measure or a number is

precise within certain percent of error or a certain number of significant digits. Effective measures are in terms of errors involved. Such errors are either positive or negative. The precision of a measure is evaluated in terms *of* apparent error. The accuracy of a measure is evaluated in terms of the relative error or percent of error made.

Pupils should possess the sense of precision in the act of measurement made. Developing a sense of precision helps to discipline pupils mind. Pupils acquire the sense of precision when they observe critically, perceive relationship, tabulate data, analyse results and arrive at correct & precise inferences or conclusions. There is a need to develop skills in using measures and estimating quantities and magnitudes without measuring and to gain appreciation of the role of measurement. In modern life by introducing sound courses of mathematics in school stages.

1.1.4 Abstractness

Mathematics is mostly theoretical and difficult to understand. Mathematics is characterized by geometric and formalized attributes or otherwise by non-representational qualities without reference to specific circumstances on practical experience. It has no reference to material objects or specific examples. Mathematics is not concrete but sometimes the abstract mathematical concepts can be explained through concretization and. practical activities. For example, the concept of 'number' in mathematics is abstract since it does not refer to any specific quantity; for instance the number five does not refer to live objects [marbles/books, students ...].The symbols [5, V,] represent quantitive measure of five. Five simply means the fiveness of five the 'representative ness' of five objects; which is an abstract concept of five.

'Fiveness' of five objects thus, representative ness of five/5/V is a cardinal concept of number which is an 'abstract concept' which refers to fiveness of five objects. 'The concept of 'fiveness' of five objects can be compared with of 'whiteness' of paper or wall, egg or idly, which is an abstract quality. The obvious illustrations are to appreciate the fiveness of five students, five pens ... etc. Similarly to understand the division into seven equal parts of Rs.14,35cms or 56 ones is the mental functioning of identifying mathematical relationships between the elements of a set or any other. All the numbers and operations such as 5,6... and union [u] and intersection [n] are abstractions; percent, length, volumes are abstractions; sum, difference, product, average are facts that concern elements or aspects which may appear with countless different concrete surroundings. Concretization helps to develop abstract thinking while learning mathematics. When concrete experiences are not possible, the mathematical concepts, principles well understood by students serve as concrete foundation for further abstract level of mathematic learning.

1.1.5. Symbolism

Mathematics is a language. It is a technical way of expressing a certain range of ideas in a form which is convenient for applying reasoning processes. The language of mathematics is applicable to a limited range of ideas with comparative degree. It has its own precision, conciseness an accuracy which are absent from any other language.

A language of mathematics is much more than the word, symbols and the grammar it contains. The way all these components are put together provides a complete logical reasoning and thinking with which the concept principles and rules of mathematics are associated. Pupils learning mathematics should be able to distinguish the uses of symbols; because they represent basically three different kinds of meanings and understanding; namely

1. The elements or mathematical object: they are "numbers represented by the symbols [numerals]; e.g.5,12; sets such as A, B, or geometrical objects such as lines 1, m which are used in

mathematical sentences like 5< 12; ACB = 0,1/m etc.

2. The relationship between elements: The symbols like >, =, < etc. represent the verbs of mathematics. These express the particular way in which the elements are related to each other.
3. The operation on elements: The symbol; +,x, u, n ...represent operations performed on elements such as addition, multiplication, union and intersection respectively and brackets or parenthesis perform the function of punctuation in the language of mathematics as could be seen from the different answers for

   81 ^ [27^ 3] =9 and [81 4- 27] H- 3 = 1

   Thus, it is essential to insert the appropriate mathematical punctuation the bracket or parenthesis.

1.2. MATHEMATICS AS A SCIENCE OF MEASUREMENT AND QUANTIFICATION

We all know that mathematics is defined in simple terms as the science of quantity, measurement and spatial relationships. It is a systematized, organized and exact branch of sciences. It deals with quantitative facts, relationships as well as with problems involving space and form. The dictionary meaning of mathematics is that it is either the science of number and space or the science of measurement, quantity and magnitude also the logical study of shape, arrangement and quantity.

1.3. MATHEMATICS AND ITS RELATIONSHIP WITH OTHER DISCIPLINES

Leaving aside Mathematics, languages, physical education and work experiences all the other subjects taught in the high school classes can be classified into two groups i.e. subjects related to arts group in which History, Geography, Civics, Economics can be included and the subjects related to science group in which we can place Physics, Chemistry, Astronomy, Botany, Zoology etc. Here the question may arise: in which group Art or Science should Mathematics be placed. The answer is difficult because Mathematics has its connection and roots in the Science as firmly as it has with the Arts subjects. It nourishes and in turn gets nourished from Science as well as Art. That is why it is termed as Science of all Sciences and Art of all Arts.

Whatever may be the form of the society it has an educational structure for realizing certain aims and objectives. Different subjects of the curriculum help in the realisation of these set goals. Although the courses are different yet they have common goals. This commonness draws them nearer and in this way learning in .a particular subject affects the learning of other subjects and vice versa. But here Mathematics plays a more specific role in helping the learning of other subjects. It has direct or indirect relationship with almost all the subjects. Let us begin with the relationship of Mathematics with Science.

Relationship of Mathematics with other Disciplines

1.3.1. Mathematics and Physics

If we take physics we see that its study requires the knowledge of Mathematics at every point. All the physical laws, laws of motion, laws of lever and pulleys, laws of refraction and reflection, laws of magnetization, laws of electric current, movement of the earth and planets and laws of quantum energy can only be understood and applied with the help of the understanding of Mathematics. The need of the numerical calculations in dealing with the problems in Physics clearly reveals the value of Mathematics in learning Physics. The lenses and other equipments used in microscope, telescope, photographic camera movie can only be made useful and workable with the help of intensity, power and arrangement decided by the basic principles of Mathematics. In this way what we study in 'Physics' can only be studied effectively with the proper use of Mathematics.

1.3.2. Mathematics and Chemistry

Study in chemistry is also helped by the knowledge of Mathematics. The compositions and properties of the different elements in Chemistry can only be understood properly with Mathematics. For example the type of composition, no matter whether volumetric or graviometric is decided by the laws of ratios and proportions governed by Mathematics. The study of compounds, mixtures, laws of chemical combination and the study of molecular or atomic structures, chemical names of formula and chemical equations all are based on the laws of Mathematics. In the preparation of different gases and chemical products like bleaching powder, salts, acids, medicines and other day to day use products, we need exact measurements in terms of weight, ratios and other calculations. In this way it is no exaggeration in saying that Mathematics is used in the study of chemistry right from petty chemical reaction to the preparation of chemical fuels of modern rockets and bombs.

1.3.3 Mathematics and Botany-Zoology

In all the experiments and studies of Botany and Zoology, we take the help of Mathematics. The cellular construction of animals and vegetables, heredity, process of reproduction, balanced diet and similar other topics need the knowledge of Mathematics. In any organism if we try to study the anatomical structure and pattern of definite growth and development, we have to take the help of the subject Mathematics. The graphs and statistical concepts used in these branches can also reveal the need of Mathematics or Geometry.

1.3.4. Mathematics and Astronomy

Astronomy in one sense is totally based on the learning of Mathematics. The complicated and intensive study connected with the movements of planets and satellites, their relative attraction and distances and study of their orbits can only be possible with the knowledge of Mathematics. For an ordinary individual it would be a great wonder to know that eclipses, tides, and the rising and the setting of planets and starts happen at a fixed day and time but for a student of Astronomy it is a usual phenomenon conducted through the rules of Mathematics. In this way Mathematics does not only help in the understanding of Astronomy but also renders the astronomers a reliable help in the realization of their dreams to walk straight on the distant planets like Venus and Mars.

1.3.5. Mathematics and Medical Science

In medicines the diagnosis as well as remedial treatment is based on the knowledge of Mathematics. Temperature, blood pressure, deficiency and excess of the minerals and other substances, the pressure or absence of undesirable substances and parasites in the blood, urine and stool tests can only be detected and correctly measured with the adequate knowledge of Mathematics. In preparation of the doses of medicines one has to take it to account the mechanism of measurement which is not possible without Mathematics. Can we imagine that particular ingredient of the medicine may prove most fatal or injurious to a person if its ratio or quantity is increased or decreased a little. Both nurses and compounders will feel handicapped in the proper look after of the patients and preparation of the proper doses of the mixtures and medicines if they happen to be ignorant of Mathematics

1.3.6. Mathematics and Engineering

Mathematics is the base of all the Engineering, Surveying and measurements which help the Science of Engineering to construct large bridges, plan the net work of canals and dams, extend railway lines across the wide forests and lofty mountains, control the floods and establish the heavy

industry. Wise engineers with the help of the knowledge of applied mathematics at command, are always in a position to serve the society and country in any front of Engineering - mechanical, electrical, civil etc.

In this way we can realise that Mathematics is quite indispensable in learning Science Subjects. Actually the relationship between Mathematics and Science is just like the relationship between the body and its soul. Body [Science] has no meaning without its soul [Mathematics]. Soul may have its existence without body. But in true sense, the existence of the soul will prove fruitful only when it carries body along with it. In the same way Mathematics or Science cannot bring any fruitful result if they are not integrated and used in combination. Whatever we see in the modern world of Science and technology has its root in the progress and improvement of Mathematics. That is why Bacon has said that "Mathematics is gateway and key of Sciences". A student of Science should therefore, try to learn Mathematics for gaining adequate success in the field of sciences.

1.3.7 Mathematics and History

History is nothing but the systematic study of the past events which requires the knowledge of Mathematics for its exact description and interpretation. A historical record, however, important it may be, is meaningless if it does not carry the adequate concept of time and this concept can only be understood with the knowledge of Mathematics. History will merely tell a student that Alexander the Great invaded India in the year 326 B.C. where Mathematics would help him to realise the total time passed to the occurrence of the said event in relation to the present, running year. In the same way the extent, organisation and duration of the different empires of the past can only be understood and appreciated with the knowledge of Mathematics. A student of History who has to acquire the knowledge about time line, dynasty and historical maps can be helped by the knowledge of Mathematics at his command.

1.3.8 Mathematics and Geography

Geography is nothing but a scientific and mathematical description of our earth in its universe. We can quote so many examples from the subject Geography that its proper learning is only possible with the adequate knowledge of Mathematics. The dimension and magnitude of the earth, the formation of days and nights, change of seasons, lunar and solar eclipses, tides, currents, movement of winds, falling of rain, factors influencing the climate of a region etc. are so many learning areas of Geography which needs the knowledge of Mathematics. A student of Geography is required to possess sufficient knowledge of drawing and understanding the maps. Study of Mathematics may certainly help him in this task. He can locate and describe the position of a particular place in the world map. He can also understand and calculate the local, standard and international time with the help of the knowledge of Mathematics.

1.3.9 Mathematics and Economics

In learning Economics also the students use the language and knowledge of Mathematics. The production, sale and purchase and distribution of commodities can only be regularised and maintained by the help of Mathematics. The rate of exchange of different currencies belonging to different nations are always decided by Mathematics. Whether we talk of export and import or value of currency and international business relationship of a country or we talk of the internal economic structure in terms of the budget preparation, planning and tax-collection of a country. We have to take help from the terms used in the study of Economics like de-valuation of the currency, long term saving schemes; Labour and capital relationship can be understood with the help of Mathematics. The whole commercial system, Banking and Insurance etc. gets its nourishment from Mathematics. In any

economic planning one has to collect different types of statistics. These statistics can only be collected, maintained and interpreted with the help of the knowledge of Statistics—an offshoot of Mathematics. The language of Economics in terms of different types of tables and graphs is nothing but the language spoken by Mathematics. It needs its own dictionary in terms of the knowledge of averages, ratio and proportion, formulae, discount, interests and the necessary geometrical as well as statistical concepts.

1.3.10. Mathematics and Psychology

Today in the field of Psychology, statistical methods are occupying very important place and therefore the study of Mathematics has become a necessary for gaining adequacy in Psychology. In this connection Herbert Spencer has rightly said that *"It* is *not only possible but necessary that Mathematics should be applied to Psychology".* When a student of Psychology wants to study the human behaviour with adequate precision and objectivity he has to take the help of Statistics. Sometimes he has to calculate the measure of central tendency in terms of mean, median or mode and at the other time he has to compute coefficient of correlation, standard deviation and other Statistics for deriving valid and reliable inferences. In all these tasks one has to take the help of Mathematics and hence Mathematics is indispensable in any study concerning Psychology.

1.3.11. Mathematics and Languages

The key of learning any language—regional, national, or international, lies in its grammar. In grammar we apply some set rules and principles governed by Mathematics. Where to apply comma, full stop and put question mark is decided as exactly as we decide to write 10 after 9. Nouns, pronouns, verbs, adjectives and other forms of sentence are used at their appropriate set places. Not only the prose but the poetry also follows the rules of Mathematics. The aesthetic value of the poetry can only be enjoyed if it is decorated with the regularity, symmetry and precision of Mathematics. Mathematics has its own language in the form of numbers, signs, symbols, formulae and equations. This language of Mathematics is borrowed by languages for bringing objectivity, precision, exactness and accuracy in communication and expression of ideas and thoughts.

1.3.12. Mathematics and Drawing

Drawing and Mathematics go side by side as far as acquisition of the knowledge in any of these areas is concerned. In drawing any picture, portrait, model or design one needs the knowledge of Mathematics in terms of geometrical forms and skills as well as arithmetical measurement and computation. In the same way the topics of area, menstruation as well as geometrical propositions and constructions in Mathematics need the drawing skills for their proper learning. That is why in any scheme of learning Mathematics or Drawing integration of these two subjects is properly cared for.

1.3.13. Mathematics and Music and Dance

The sound systematized and organized on some set Mathematical principles is known as Musical sound. All the musical instruments—harmonium, violin, guitar, sitar, flute etc. follow the laws of mathematics. Which particular 'Raga' has what type of rhythmical order, the ascending order of tone and pitch all need the knowledge of Mathematics. In the art of dance also the movements of the limbs and postures require mathematical rhythm and tuning.

1.3.14. Mathematics and Physical Education

In Physical Education one has to learn about Physiology and Hygiene. The knowledge about the body systems requires mathematical calculations. Similarly to know about temperature and blood pressure of the body, pulse rate, balanced diet for the different kinds of individuals, the student has to take the help of Mathematics. In games and physical activities also, the learning of essential skill requires an adequate knowledge of Mathematics because everywhere one has to use weighing, counting and

measuring as standard setting and evaluating devices.

1.3.15. Mathematics and Work Experience Activities

For integrating Education with work, to develop a positive attitude towards manual work and increasing productivity a subject work-experience is being included in the Secondary School Curriculum. The students are asked to choose one or two areas such as vegetable growing, soap making, some useful direct experiences. Gaining experiences in such areas also requires the knowledge of Mathematics. For example soap or Ink making requires a set procedure based on measurement governed by Mathematics. In floriculture or vegetable growing also one cannot do without Mathematics. Right from surveying, measuring and distribution of land to the ploughing, manuring, watering, taking care of plants and selling of the produced goods one needs the knowledge of Mathematics. Similarly clay modeling, woodwork, metal work, cane-work and leather work all require the knowledge of Mathematics for purchasing raw material, processing finished products and disposing them properly by getting adequate economic and educational gains. In a nutshell in all the areas of work experiences wherever the task of computation and measurement is asked for, the knowledge of Mathematics is indispensable.

As teachers of mathematics we should encourage our students to solve the problems by logical reasoning or otherwise and be ready to help them if they have difficulties, individual help and general class discussion if necessary would be fruitful.

It becomes crystal clear from the above discussion that Mathematics occupies a key place in the school curriculum. It gives language to the languages, artistic touch and beauty to the arts, scientific essences to the sciences and movement to the work experiences activities of the students. It appears that Mathematics is a life blood of all the activities going inside a school. If we take different subjects and activities as different pearls, Mathematics may be compared with that golden chain which fixes firmly many pieces of different pearls [other subjects of the curriculum] to form a beautiful necklace. In a necklace where the chain is essential for every piece of the pearls, the pearls are also very much essential for magnifying the importance and usability of the chain.

1.4. CHARACTERISTICS OF A GOOD MATHEMATICS TEACHER

- A good mathematics teacher should have an extensive knowledge and love of mathematics.
- A good mathematics teacher should have good content knowledge of Mathematics.
- He/she needs to have a profound understanding of basic mathematics and to be -able to perceive connections between different concepts and fields.
- A good mathematics teacher needs to understand pupil's thinking in order to able to arrange meaningful learning situations.
- A good mathematics teacher needs to be able to use different strategies to promote pupil's conceptual understanding.
- A good mathematics teacher needs additional pedagogical knowledge the ability to arrange successful learning situations, knowledge of the context of teaching and knowledge of the goal of education.
- A good mathematics teacher's beliefs and conceptions should be as many-sided as possible and be based on a constructivistic view of teaching and learning. The view of mathematics viz., knowledge, beliefs, concepts, attitudes and emotions and about on self as a learner, as a teacher of mathematics.

- A good mathematics teacher should know:
  - What is Mathematics?
  - How is Mathematics taught?
  - How is mathematics learned?
- A good mathematics teacher should posses an endless amount of patience because there as many different ways that students actually learn mathematics. And they learn at many different speeds.
- A good mathematics teacher should understand Piaget's theory on how youngsters create logic and number concepts is time well spend for mathematics teacher.
- *A* good mathematics teacher should have strong classroom management skills.
- A good mathematics teacher should be able to explain mathematics well.
- A good mathematics teacher should have the ability of making mathematics relevant and incorporating real-world examples.
- A good mathematics teacher should have a good rapport with pupils.
- A good mathematics teacher should focus on concepts as well as procedures.
- A good mathematics teacher should help students when they are struck by showing them how to do problems.
- Good mathematics teacher needs the ability to do quick error analysis, and must be able to concisely articulate what a student is doing wrong, so they can fix it.

1.5. Contributions of Some Indian Mathematicians to the development of Mathematics.

1.5.1. Aryabhata and his contributions

In the history of Indian mathematics, Aryabhata is a very respectable name. Very often it is being repeated. Thereby it is difficult to ascertain how many mathematicians of this name had been in ancient India. However, it is quite certain that there were at least two Aryabhatas. One of them was born in 476 A.D. at Kusumapura, the city of flowers [Patliputra] near present city of Patna in Bihar and wrote the book Aryabhatiya, is known as "Aryabhata first". The other mathematician bearing the same name who wrote the book "Maha Aryn Siddhant" in 950 A.D. is known as Aryabhata Second. The period of Aryabhata first or the elder Aryabhata has been the golden period of Indian mathematics. Let us try to know something about the life and work of this great mathematician.

Scholars differ about the birth place of the elder Aryabhata. He worked and lived at Kusumpur but it does not prove that it was his birth place. Some writers claim that he belonged to Kerala and in support of this claim they assert that the Calendar System invented by Aryabhata is still prevalent there. Whatever may be his birth place, it is quite certain that he rose to fame while working at Kusumpur. Here at the age of 23 years he wrote Aryabhatiya the first Indian astronomical text to contain a section devoted entirely to basic mathematics.

Aryabhatiya in fact, is a small publication of 121 shloks [verses].

This work of Aryabhata shows his greatness, originality and creativity in the field of

mathematics by bringing into light some of his following contributions :-

1. Aryabhata invented a notation system consisting of alphabet numerals. Digits are denoted by alphabet numerals in this system. Devnagri script contains Varga letters [Consonants] and Avarga letters [Vowels]. Digits from 1 to *25* are denoted by the first *25* varga letters.
2. Although earlier to Aryabhata, the method of extracting square root was evolved by Jain mathematicians, yet Aryabhata is known for giving its simple and clear explanation. He writes as :-

   "One should always divide the avarga by twice the [square] root of the [preceding] varga. After subtracting the square [of the quotient] from the varga, the quotient will be the square root to the next place."
3. Aryabhata put more appropriate uses of the decimal system.
4. He gave almost all the formulae for knowing area of different figures like area of a square, rectangle, triangle, rhombus circle, and volumes of sphere and cone etc. He also tried to point out the construction of different geometrical figures—triangle, quadrilateral, circle. It shows how much interested he was in the practical geometry.
5. Not only in arithmetic but in algebra also. Aryabhata contributed a lot. In Aryabhatiya he has given the method

   of addition, subtraction, division and multiplication of simple and compound algebraic
6. He tried to give a rule for summing an arithmetic series after the pth term.

"The desired number of terms minus one, halved plus the number of terms which precedes, multiplied by the common difference between the terms, plus the first term, is the middle term. This multiplied by the number of terms desired is the sum of the desired number of terms or the sum of the first and last terms is multiplied by half the number of terms."

[where a and *1* are the first and the last terms of the progression, *d* the common difference between terms and n being the number of terms extending the[p+ l]th to the [p+n]th terms in arithmetical progression.

7. One can imagine the intelligence and work of Aryabhata know that he tried to solve indeterminate linear equations like ax ± by = c by the method of the continued fractions which is substantially the same as the method in use today.
8. The identities like the following are found in Aryabhatiya for the first time in the history of mathematics.

$$I^2 + 2^2 + \ldots\ldots\ldots + n^2 = l/6[n\,[n + l][2n + 1]]$$

$$l^3 + 2^3 + \ldots\ldots\ldots\ldots + n^3 = [1+2+\ldots\ldots + n]^2$$

In this way 'Aryabhata was much ahead of his time. He dared to begin a new chapter in the development of mathematics and astronomy through his valuable researches and contributions. Surely he has an irremovable place in the history of mathematics.

1.5.2. Bramgupta and his contributions

Brahmgupta is known as one of the most distinguished, and prominent mathematician of the ancient India. He was born in 598 A.D. in Sindh province [now in Pakistan]. His father's name was Vishnugupta. According to Smith he later on began to live and work, in the great astronomical centre Ujjain [now in Madhya Pradesh]. At this place he wrote valuable books on Astronomy and-Mathematics. Among them few like Brahm-Sphuta Siddhanta, Khand Sadhak and Dhyan

Grahopdesh are quite popular.

'Brahm-Sphuta-Siddhanta, which was written by Brahmgupta when he was only 30 years old, contains 21 chapters. The most useful chapters are 12th and 13th which are named as Ganitadhya and Kutakhadhyaka. Ganitadhya deals with arithmetic and geometry whereas Kutakhadhyaka deals with algebraic problems. The other remaining chapters deal mostly with astronomical knowledge. This book Brahm Sphuta-Siddhanta helped Arabs to acquaint with the Indian Astronomy. Khalifa Abbasial Mansoor [712-775 A.D.] called astronomer Kanka of Ujjain for explaining Brahm-Sphutat-Siddhanta to the astronomers of his country and made the book translated into Arabic language.

Brahmgupta contributed a lot to almost all the branches of mathematics. The following information may reveal this fact

Contribution in Arithmetic

1. The method of squaring first occurs in Brahm-Sphuta-Siddhanta. Concisely it has been written as :-

   "Combining the product twice the digit in the less [lowest] place into the several others [digits] with its [i.e. of the digit in the lowest place] square [repeatedly] gives the square."

   He also gave method of cubing and extracting square roots as well cube roots. About square root he writes, "The pada [root] of a kriti [square] is that of which it is the square."

2. He also explained clearly the operations of addition, subtraction, multiplication and division with different types of fractions. For example for the multiplication of fractions he writes:

   "The product of the numerators divided by the product of the denominators is the [result of] multiplication of two or more fractions."

3. The method of inversion called 'vilomgati' [working backwards] also for the first time was completely explained by Brahmgupta. He writes:

   "Beginning from the end, make the multiplier divisor, the divisor multiplier; [make] addition, subtraction and subtraction, addition, [make] square, square root and square root, square ; this gives the required quantity."

4. He contributed a lot in understanding the concept of zero. He defines it as a - a = 0. In. addition to this he also explained operations of addition, subtraction, multiplication and division with zero but he fell into the error of assuming that "cipher [zero] divided by cipher [zero] is cipher [zero]."

Contributions in Algebra

He, for the first time, treated algebra as a separate branch by dealing it separately in Kutakhadyaka. His main contributions in this branch are as follows:

1. Brahmgupta's work on indeterminate equations displays his greatest

   power. Aryabhata had indicated a method of arriving at a solution of the indeterminate equation of the first degree. Brahmgupta went ahead by giving a complete integral solution of the equation *ax +by -[a.* band c being integral] and by his elaborate treatment of the indeterminate equation $ax^2+l=y^2$

2. He was the first Indian writer who applied algebra to astronomy of any great extent. As illustration the following problem may be cited

3. "A bamboo 18 cubits high was broken by the wind, its top touched the ground 6 cubits from the foot. Tell the length of the segments of the bamboo."

1.5.3. Bhaskaracharya and his contributions

The history of Indian mathematics has been illuminated twice by the name Bhaskaracharya. Bhaskaracharya first, was born a few years after the Aryabhatta first. He was a good student as well as critic of mathematics and astronomy. The other Bhashkaracharya, known as Bhaskar II was born many centuries after the first. He has been the most powerful and creative mathematician India ever produced. In the following pages we would try to focus our discussion on this Bhaskaracharya.

Bhaskaracharya served as head of the astronomical observatory at Ujjain where Brahmgupta had served in a similar capacity some five hundred years ago. According to his own book "Siddahanta-Shiromani" he was born in 1114 A.D. in the village Bijjada Bida [at present situated in Bijapur-Mysore State] near Shahyadri mountain. He came of an old learned family of some nobility and title, possessing an established tradition of scholarship. The name of his father [as well as Guru teacher] was Maheshwar. The book Siddhanta Shiromani was written in 1150 A.D. at the age of 36 years. Four chapters of this book have survived. They are Lilavati, Vijaganit, Goladhyaya and Grahganit. Why did he name the first chapter as Lilavati, is disputable. Some say it is because of the name of his daughter Lilavati, the others consider it a mere style of writing prevalent in those days. Whatever may be the reason for the naming, Lilavati is a valuable treatise, having 278 verses.

In addition to Siddhant Shiromani, Bhaskaracharya, wrote other valuable books like Karan Kotuhal, Samiaya Siddhant Shiromani. Goladhayay Rasguna, Surya Siddhant. The original texts of these works are however, missing.Through his valuable writings, Bhashkaracharya contributed a lot in the field of mathematics and astronomy. Some of his important contributions are mentioned below.

1. Bhaskar for the first time brought the idea of infinity while dividing a number by zero. He writes, "The fraction of which the denominator is cipher is termed as infinite quantity. In this quantity there is no alteration, though it may be inserted or extracted ; as no change takes place in the infinite and immutable God at the period of destruction or creation of worlds, although numerous orders of being are absorbed or put forth."
2. He also contributed much in the field of mensuration. He gave many important formulae for the computation of the area and volumes of different figures like the following :-

   Area of a sphere = 4 ' area of a circle

   Volume of a sphere = area of a sphere 'l/6of its diameter
3. Bhaskaracharya also dealt with cubic equations and Biquadratic equations in his writings. The following types of problems are available in the 2nd chapter of Siddhant Shiromani.

   $x^4 + 12x = 6x^2 + 35$ [Biquadraticequation]

   $x^3 - 2x^2 - 400x = 9999$ [Cubic equation]

   It reveals that he was quite ahead in this field.
4. Bhaskara's greatest strength lay in his ability to handle problems which lead to indeterminate equations. An attempt is here made to arrange the various problems in the order in which they would appear in a modern treatise.

   [i] $ax+c=by$

   [ii] $ax+by+cz = d$

   [iii] $ax+by+d = xy$

Equation of the types considered above were fairly common among earlier writers. But Bhaskara advanced well beyond his predecessors in his approach to indeterminate equation of the second degree like the equation $ax^2 + 1 = y^2$, the so called Pell's equation.

5. Bhaskaracharya is known for his poetic presentation of the complicated and abstract problems of mathematics.
6. Bhaskaracharya was much ahead of his time in so many aspects. In Goladhayaya he gives the following evidence in support of the roundness of the earth.

7. "The hundredth part of the circumference of a circle seems, to be a straight line. Our earth is a huge sphere, we can see only a small fraction of it; therefore it appears flat.
8. He also had the knowledge of gravitational power long before Newton. He named it as Dharnikatmak Shakti He explains it as below :-

"All things appear to fall on the earth simply because they are attracted by it on account of its Dharnikatmak Shakti [gravitational power].

In this way what has been contributed by Bhashkaracharya is unaccountable. In every sense he was a celebrated astronomer and mathematician. He shines like sun [Bhaskar] in the world of mathematics.

1.5.4. Shrinivasa Ramanujan and his Contributions

One day a primary school teacher of the third form was telling to his students, "If three, fruits are divided among three persons, each would get one. Even if 1,000 fruits are divided among 1,000 persons, each would get one." From this he tried to generalize that any number divided by itself was unity. This made a child of that class jump and ask, 'Is zero divided by zero also unity ? If no fruits are divided among nobody, will each get one?"

Do you imagine who was this little child? He was Shrinivasa Ramanujan, the wonderful young Indian mathematician of the 20th century who was so intelligent that as student of class III of a primary school he successfully worked out the properties of the Arithmetical, Geometrical and Harmonic progression and up to class IV he almost solved all the problems of the Loney's Trigonometry meant for the degree classes.

This extra-ordinary mathematician of his time was born in a poor Brahmin family on 22nd December 1887 at Erode in Tanjore district of Madras State. His father Srinivasa Ayyangar was an accountant at a cloth merchant at KumbkaKonam. While his mother, Komalammal, was the daughter of a petty official [jamin] in the district Munsif's court at Erode.Ramanujan got much of his earlier education in the town high school at Kumbka Konam. He always stood first in his class and got scholarship. He was very much popular for his interest and extra-ordinary abilities in mathematics He was so bright that he was declared child mathematician, at the age of 12 by his teachers. He used to entertain his friends with theorems and formulae, with the recitation of complete list of Sanskrit roots and with repeating the value of pi and the square root of two to any number of decimal places. In the year 1903, when he was 15 and in the sixth form at school, a friend of his gave him a book "Carr's Synopsis of Pure and Applied Mathematics" from the library of the local government college. It was this book that awakened and stimulated his genius. He verified many of the results in the book and discovered many new results of his own. Besides engaging in this original work he did not miss his regular studies and as a result gained a place in the first class in the matriculation examination of the University-of Madras held in December 1903. This enabled him to secure Subramaniam Scholarship

and join the F.A. [First examination in arts] class in the Government College, Kumba Konam. Owing to weakness in English, for he gave no thought to anything but mathematics, he failed in his next examination and lost his scholarship. He then left Kumbka Konam, fast for Vizagapatam [Andhra Pradesh] and then for Madras. He resumed his studies, completed his second year course in the Pachiappa College in 1906. Unluckily he got ill at the time of examination and therefore appeared for the university examination in December 1907 but at this time also he got failed, and then determined not to try again.

For the next few years he continued his independent work in mathematics. In 1909 he was married to Janaki and it became necessary for him to find a job of permanent nature. In the course of his search for work, he was got introduced to a true lover of mathematics, Diwan Bahadur Ramchandra Rao. For some months he was supported by Shri Ramachandra Rao. Then he accepted his appointment as a clerk in the office of the Madras Port Trust. While working as a clerk he never slackened his interest in mathematics. He made his one of the works published in the Journal of the Indian Mathematical Society in 1911 at the age of 23. He wrote a long article on "Some properties of Bernoulli's numbers' in the same year. In 1912 he contributed two more notes to the same journal and also several questions for solution. Meanwhile he began correspondence with Professor G.H. Hardy, a leading mathematician of his time. To his first letter he attached 120 theorems of his own creation.

At last in May of 1913, as the result of the help of many friends, Ramanujan was relieved of his clerical post and was given a special scholarship. Hardy made efforts to bring Ramanujan to Cambridge and helped him to learn modern mathematics so as to acquaint him with all the up-to-date development in the field of mathematics. In 1916 he got honorary B.A. degree of the University of Cambridge. About making Ramanujan to learn at ambridge. Hardy writes "It was impossible to ask such a man to submit to systematic instruction, to try to learn mathematics from the beginning once more I had to try to teach him and in a Measure succeeded though obviously learnt from him much more than he learnt from me".

In the spring of 1917, Ramanujan first appeared to be unwell. He went to nursing home at Cambridge in the early summer and was never out of bed for any length of time again. For brief period he resumed some active work, stimulated perhaps by his election to the Royal Society and Trinity Fellowship. Due to Tuberculosis he left for India and died in Chelpet, Madras on account of this disease on April 26, 1920 at the age of thirty three.

In his short span of life he contributed so significantly in the field of mathematics that many eminent authorities accepted him "quite the most extraordinary mathematician of his age." Some of his achievements are enumerated below

1. Divergent Series: His first investigation in this direction was sent to Professor Hardy in form of 120 theorems in the year 1913. Commenting on the merit of these theorems, Hardy wrote, "I had never seen anything the least like them before. A single look at them is enough to show that they could only be written down by a mathematician of the highest class"
2. Hyper geometric series and continued fractions: He was unquestionably one of the greatest masters in this field. Commenting on this Hardy writes, "It was his insight into algebraic formulae, transformation of infinite series and so forth that was most amazing. On this side, most certainly I have never met his equal and I can compare him only with
   Euler and Jacobi"
3. Definite Integrals: He produced quite a number of results in this field in the form of general

formulae. These are all included in his three quarterly reports to the University of Madras.

4. Eliptic Functions: He tried to handle eliptic functions profusely commenting on his ability in this direction Hardy writes, 'Ramanujan shows at his very best in the parts of the Theory of Elliptic Functions allied to the Theory of Partitions."

5. Partition Functions: Before Ramanujan very little was known about the arithmetical properties of a partition function P [n] where n is odd or even. Ramanujan was the first, and till his time, the only mathematician to discover any such properties. For the first time in 1917, Hardy and Ramanujan, jointly examined, the question of how large the number of partitions of n is, when n itself is large. They gave the answer in the form of an asymptotic series and also estimated the error involved in taking a definite number of terms only.

6. Fractional Differentiation: The insight and generalizations derived in this field by Ramanujan are quite wonderful. He gave a meaning to Eulerian Second Integral for all values of n negative, positive and fractional.

In fact Ramanujan had a very original and intuitive approach to numbers. It appears that every integer was one of his personal friends. In the simplest array of digits he detected wonderful properties, congruence's, symmetries and relationships which had escaped the notice of even the outstanding gifted theoreticians. Hardy recalls a meeting of Mr. Little-wood with Ramanujan. He had taken a taxi cab no. 1729 and remarked that the number seemed to him rather a dull one. No, Ramanujan replied. "It is a very interesting number, it is the smallest number expressible as a sum of two cubes in two different ways."

$[1729=l^3+2^3=9^3+10^3]$

In this way one can judge the merit and competency of Ramanujan as a first rate mathematician. Although he had not got enough opportunity for college education in the subject mathematics and most of his time was spent either in struggling for the means of livelihood or fighting his ill health he contributed a lot in the field of mathematics by showing his profound and invincible originality. He was a mathematician whom only first class mathematicians can follow and it is not surprising, therefore that he attracted little attention outside his profession. But his work has left a memorable imprint on mathematical thoughts.

1.5.5. Euler and his Contributions

Leonhard Euler

The 18th-century Swiss mathematician Leonhard Euler [1707-1783] is among the most prolific and successful mathematicians in the history of the field. His seminal work had a profound impact in numerous areas of mathematics and he is widely creed for introducing and popularizing modern notation and terminology, particularly in analysis.

Mathematical Notation

Euler introduced much of the mathematical notation in use today, such as the notation f[x] to describe a function and the modern notation for the trigonometric functions. He was the first to use the letter 'e' for the base of the natural logarithm, now also known as Euler's number. The use of the Greek letter <3 to denote the ratio of a circle's circumference to its diameter was also popularized by Euler [although it did not originate with him]. He is also creed for inventing the notation i to denote -

Complex Analysis

Euler made important contributions to complex analysis. He discovered the scientific notation. He discovered what is now known as Euler's formula, that for any real number q, the complex exponential function satisfies

$$e = \cos G + z \sin G$$

This has been called "the most remarkable formula in mathematics " by Richard Feynman. Euler's identity is a special case of this:

$$e^{™+1} = 0$$

This identity is particularly remarkable as it involves e, 3, i, 1, and 0, arguably the five most important constants in mathematics.

Analysis

The development of calculus was at the forefront of 18th century mathematical research, and the Bernoullis—family friends of Euler—were responsible for much of the early progress in the field. Understanding the infinite was naturally the major focus of Euler's research. While some of Euler's proofs may not have been acceptable under modern standards of rigor, his ideas were responsible for many great advances. First of all, Euler introduced the concept of a function, and introduced the use of the exponential function and logarithms in analytic proofs.

Euler frequently used the logarithmic function as a tool in analysis problems, and discovered new ways by which they could be used. He discovered ways to express various logarithmic functions in terms of power series, and successfully defined logarithms for complex and negative numbers, thus greatly expanding the scope where logarithms could be applied in mathematics. Most researchers in the field long held the view that $\log[x] = \log[-x]$ for any positive real x since by using the additivity property of logarithms $2\log[-x] = \log[[-x]^2] = \log[x^2] = 2\log[x]$. In a 1747 letter to Jean Le Rond d'Alembert, Euler defined the natural logarithm of -1 as *id* a pure imaginary.

In addition, Euler elaborated the theory of higher transcendental functions by introducing the gamma function and introduced a new method for solving quartic equations. He also found a way to calculate integrals with complex limits, foreshadowing the development of complex analysis. Euler invented the calculus of variations including its most well-known result, the Euler-Lagrange equation.

Euler also pioneered the use of analytic methods to solve number theory problems. In doing so, he united two disparate branches of mathematics and introduced a new field of study, analytic number theory. In breaking ground for this new field, Euler created the theory of hypergeometric series, q-series, hyperbolic trigonometric functions and the analytic theory of continued fractions. For example, he proved the infinitude of primes using the divergence of the harmonic series, and used analytic methods to gain some understanding of the way prime numbers are distributed. Euler's work in this area led to the development of the prime number theorem.

Number Theory

Euler's great interest in number theory can be traced to the influence of his friend in the St. Peterburg Academy, Christian Goldbach. A lot of his early work on number theory was based on the works of Pierre de Fermat, and developed some of Fermat's ideas.

One focus of Euler's work was to link the nature of prime distribution with ideas in analysis. He proved that the sum of the reciprocals of the primes diverges. In doing so, he discovered the

connection between Riemann zeta function and prime numbers, known as the Euler product formula for the Riemann zeta function.

Euler proved Newton's identities, Fermat's little theorem, Fermat's theorem on sums of two squares, and made distinct contributions to the Lagrange's four-square theorem. He also invented the totient function 6[n] which assigns to a positive integer n the number of positive integers less than n and coprime to n. Using properties of this function he was able to generalize Fermat's little theorem to what would become known as Euler's theorem. He further contributed significantly to the understanding of perfect numbers, which had fascinated mathematicians since Euclid. Euler made progress toward the prime number theorem and conjectured the law of quadratic reciprocity. The two concepts are regarded as the fundamental theorems of number theory, and his ideas paved the way for Carl Friedrich Gauss.

Applied Mathematics

Some of Eider's greatest successes were in applying analytic methods to real world problems, describing numerous applications of Bernoulli's numbers, Fourier series, Venn diagrams, Euler numbers, e and 5 constants, continued fractions and integrals. He integrated Leibniz's differential calculus with Newton's Method of Fluxions, and developed tools that made it easier to apply calculus to physical problems. In particular, he made great strides in improving numerical approximation of integrals, inventing what are now known as the Euler approximations. The most notable of these approximations are Euler method and the Euler-Maclaurin formula. He also facilitated the use of differential equations, in particular introducing the Euler-Mascheroni constant:

One of Euler's more unusual interests was the application of mathematical ideas in music. In 1739 he wrote the Tentamen novae theoriae musicae, hoping to eventually integrate music theory as part of mathematics. This part of his work, however did not receive wide attention and was once described as too mathematical for musicians and too musical for mathematicians.

Works

The works which Euler published separately are:

- Dissertation on the physics of sound

Mechanica, sive motus scientia analytice; expasita

- Methodus inveniendi lineas curvas, maximi minimive proprietate gaudentes
- Additamentum II [English translation]

Theoria motuum planetarum et cometarum

- Beantwortung, &c. or Answers to Different Questions respecting Comets
- Thoughts on the Elements of Bodies
- Introduction to the analysis of the infinites

1.5.6. Euclid of Alexandria

Known for "The Elements' Background

Euclid of Alexandria lived in 365 - 300 BC [approximately]. Very little is known about Euclid's life except that he taught in Alexandria, Egypt. He may have become educated at Plato's Academy in Athens, or possibly from some of Plato's students. Basically, all of the rules we use in Geometry today are based on the writings of Euclid, specifically 'The Elements'. The Elements

includes the following Volumes:

Volumes 1-6: Plane Geometry

Volumes 7-9: Number Theory

Volume 10: Eudoxus' Theory of Irrational Numbers

Volumes 11-13: Solid Geometry

The first ion of the Elements was actually printed in 1482 in a very logical, coherent framework. More than one thousand ions have been printed throughout the decades. Schools only stopped using the Elements in the early 1900s, some were still using it in the early 1980's, however, the theories continue to be those that we use today.

Euclid's book the Elements also contains the beginnings of number theory. The Euclidean algorithm which is often referred to as Euclid's algorithm is used to determine the greatest common divisor [gcd] of two integers. It is one of the oldest algorithms known, and was included in Euclid's Elements. Euclid's algorithm does not require factoring.

Euclid and His Contributions

He is famous for his treatise on geometry: The Elements. The Elements makes Euclid one of if not the most famous mathematics teacher. The knowledge in the Elements has been the foundation for teachers of mathematics for over 2000 years!

Geometry Tutorials like these wouldn't be possible without the work of Euclid.

Famous Quote: "There is no royal road to geometry."

Little is known about the life of Euclid. Although he is often referred to as 'Euclid of Alexandria', this city was not the one of his birth. We don't know exactly when he was born either, though it is known that he was in Alexandria during the reign of Ptolemy I [323-283 BC].Euclid was a Greek and before arriving in Alexandria, it is likely that he studied with Plato at his Academy in Athens.

Alexandria was a port on the northern Merranean coast of Egypt, and there Ptolemy I established an institute which was known as The Museum. We sometimes refer to it today as The University of Alexandria. Euclid was a professor of mathematics at The Museum and established the Alexandrian School of Mathematics.

Euclid is best remembered for the mathematical textbook he wrote - in fact 13 books altogether - that are collectively known as The Elements. The Elements contains 465 propositions altogether. Some of the propositions and proofs were Euclid's own work, but the book also recorded many results from his mathematical predecessors. Euclid arranged them logically into a single coherent book and used a system of mathematical proofs based on axioms.

The Elements is often thought of as a book on geometry, but it also contains number theory and some elementary algebra. Euclid also wrote at least ten other books, five of which have survived.

No copy of the original Elements is in existence today. Modern ions are based on a revision by Theon of Alexandria almost 700 years later. Nevertheless, The Elements has proved to be the most widely studied book in the world with the exception of The Bible. The Elements was used as a textbook in schools until the last century, and is still the basis of Euclidean geometry taught in modern schools.

As the most influential book on mathematics ever written, the importance of The Elements

cannot be overestimated.

One example of a contribution of Euclid to mathematics is the Euclidean algorithm for finding the greatest common divisor of two given positive integers. This is found in Book VII of The Elements.This is how the algorithm works:

1. Divide the larger number by the smaller number and find the remainder.
2. Divide the divisor [the smaller of the two original integers] by the remainder from step 1 and find the next remainder.
3. Repeat step 2 until the division is exact. The final divisor gives the greatest common divisor of the original two integers.

As an example, find the greatest common divisor of 444 and 1512.

1. 1512 -r 444 = 3, remainder 180

2. 444 -r 180 =2, remainder 84 3.180 4-84 =

2, remainder 12

84 4- 12 = 7, exactly

Now the division is exact, the final divisor [which is 12] gives the greatest common divisor of 444 and 1512.

It is not meant to be a full history of Euclid. Neither are the other articles I've written in the series 100 Great Mathematicians 'full histories'. If you want a full history, buy Euclid's biography!

1.5.7. Pythagoras and his contributions

PYTHAGORAS

Pythagoras was an ancient Greek mathematician and philosopher who was one of the most influential men in all of history. Pythagoras is one of the most famous mathematicians. Even though he was a mathematician, his contributions help all sorts of fields of study, including math, science, music and astronomy. Most of hiscontributions, and those of his followers, the Pythagoreans, are known and used throughout the world. Almost everyone knows his theorem that the sum of the squares of the lengths of the two shorter sides of a right triangle is equal to the square of the length hypotenuse.

Pythagoras was born on the Greek Island of Samos, Greece around 582 B.C. and he died around 475 B.C. He travelled around the known world visiting Babylonia and Egypt along with seeing other parts of Greece. Early on in his life several other philosophers, such as Anaximander majorly influenced him. Pythagoras was a patron of the ancient Greek Olympic games, even though he criticized them. He died at some point around the turn of the century in Metapontion

Although he made many important contributions to math we do not know much about him. We do not have anything that he wrote because he was the leader of a secret society where they didn't write down what they did. All that we have about Pythagoras are a few biographies from a long time ago.

Some accounts state that Pythagoras went to Egypt in about 535 B.C. to learn more about mathematics and astronomy. In Egypt Pythagoras visited many of the temples and talked to the priests, but he was not allowed in any of the temples except for one. In 525 B.C. the king of Persia invaded Egypt and Polycrates sent 40 ships to help the Persians invade Egypt. During the invasion

Pythagoras was captured and taken to Babylon. After Polycrates and the king of Persia died Pythagoras went back to Samos, but nobody knows how he was freed.

Pythagoras studied pure mathematics. He didn't try to solve mathematical problems. Instead, he was interested in the concept of numbers and geometric figures. He was also interested in properties of numbers; such as, odd and even numbers. He is best known for his famous geometry theorem known as the Pythagorean Theorem.

The Pythagorean Theorem is that if you have a right triangle the two sides that form the right angle are equal to the other side when each number is squared [multiplied by itself].

The equation for the Pythagorean Theorem is:

Sidel^2 + Side 2^2 = Side 3^2

For example, if you have a right triangle with sides of three, four, and five, you square three which is nine, you square four which is sixteen, and you square five which is twenty-five. Then you add nine and sixteen and you get twenty-five which is what the other side is squared.

This proves that the Pythagorean Theorem works!

You can use the Pythagorean Theorem in everyday life. It can be used on a baseball field to determine distances. Since the infield is a square with right angles you can use the Pythagorean Theorem to determine the distance from home plate on a baseball field to second base if you know the distance from first to second base.

Pythagoras also experimented with music. He found a relationship between music and math. Pythagoras found the relationship one day when he was walking past a blacksmith shop and heard the hammers hit the anvil. When Pythagoras heard the hammers hit the anvil he noticed that each hammer made a different sound according to the weight of the hammer. So, the weight of the hammer [i.e. number] decides the sound the hammer will make.

He started a religious and philosophical order known as the Pythagoreans and most of his work was completed between himself and his followers. His society was founded in Croton, in southern Italy.

The society was made up of two groups, the pupils and the learned, or teachers. These people set up the world's first organized schools.

They were a very strict order and had many restrictions, such as no meat or beans, and the new members couldn't even speak until they had listened to the teaching of their master for 5 years.

Even after the first 5 years the pupil's work was too be left anonymous, the discoveries made were either cred to the master or to the school itself.

The Pythagoreans believed that everything in the universe revolved around mathematics, such as music and the magicks. Every number had a soul and a specific meaning and value to the universe.

You of course know of my $a^2+b^2 = c^2$, but Pythagoras created this method at first just to prove that a triangle was a right triangle. Pythagoras also discovered a formula to find out how many degrees there are in a polygon. Pythagoras came up with [n-2]180°= the number of degrees in a polygon, where n represents the number of sides in the polygon. For example, a triangle has three sides, 3-2=1,1x180=180, which is the total sum of all the inner angles of a triangle. Along with that Pythagoras found out that the sum of all the outer angles of a polygon is always equal to three

hundred sixty

Pythagoras did a lot of work with proportions in other fields. Pythagoras looked at and showed the difference in pitch in ratio to the length of string plucked. It was not as Pythagoras expected. Pythagoras found out that half way along the string is not half the pitch. Pythagoras also looked to the stars and saw that the further away a planet is from where it orbited the longer it would take to go around the sun.

Through this, Pythagoras discovered a lot of different things, many being very useful.

1.5.8 Gauss and his contributions

Johann Carl Friedrich Gauss

[Latin: Carolus Fridericus Gauss] [30 April 1777 - 23 February 1855] was a German mathematician and scientist who contributed significantly to many fields, including number theory, statistics, analysis, differential geometry, geodesy, geophysics, electrostatics, astronomy and optics.

Sometimes referred to as the Princeps mathematicoruml [Latin, "the Prince of Mathematicians" or "the foremost of mathematicians"] and "greatest mathematician since antiquity," Gauss had a remarkable influence in many fields of mathematics and science and is ranked as one of history's most influential mathematicians. He referred to mathematics as "the queen of sciences."

Carl Friedrich Gauss was born on April 30, 1777 in Braunschweig, in the duchy of Braunschweig-Wolfenbiittel, now part of Lower Saxony, Germany, as the son of poor workingclass parents. Indeed, his mother was illiterate and never recorded the date of his birth, remembering only that he had been born on a Wednesday, eight days before the Feast of the Ascension, which itself occurs 40 days after Easter. Gauss would later solve this puzzle for his birth date in the context of finding the date of Easter, deriving methods to compute the date in both past and future years. He was christened and confirmed in a church near the school he attended as a child.

Gauss was a child prodigy. There are many anecdotes pertaining to his precocity while a toddler, and he made his first ground-breaking mathematical discoveries while still a teenager. He completed Disquisitiones Arithmeticae, his magnum opus, in 1798 at the age of 21, though it was not published until 1801. This work was fundamental in consolidating number theory as a discipline and has shaped the field to the present day.

Gauss's intellectual abilities attracted the attention of the Duke of Braunschweig, who sent him to the Collegium Carolinum [now Technische Universitat Braunschweig], which he attended from 1792 to 1795, and to the University of Gottingen from 1795 to 1798. While in university, Gauss independently rediscovered several important theorems;citation needed his breakthrough occurred in 1796 when he was able to show that any regular polygon with a number of sides which is a Fermat prime [and, consequently, those polygons with any number of sides which is the product of distinct Fermat primes and a power of 2] can be constructed by compass and straightedge. This was a major discovery in an important field of mathematics; construction problems had occupied mathematicians since the days of the Ancient Greeks, and the discovery ultimately led Gauss to choose mathematics instead of philology as a career. Gauss was so pleased by this result that he requested that a regular heptadecagon be inscribed on his tombstone. The stonemason declined, stating that the difficult construction would essentially look like a circle.

The year 1796 was most productive for both Gauss and number theory. He discovered a

construction of the heptadecagon on March 30. He invented modular arithmetic, greatly simplifying manipulations in number theory. He became the first to prove the quadratic reciprocity law on 8 April. This remarkably general law allows mathematicians to determine the solvability of any quadratic equation in modular arithmetic. The prime number theorem, conjectured on 31 May, gives a good understanding of how the prime numbers are distributed among the integers. Gauss also discovered that every positive integer is representable as a sum of at most three triangular numbers on 10 July and then jotted down in his diary the famous words, "Heureka! num = A + A + A." On October 1 he published a result on the number of solutions of polynomials with coefficients in finite fields, which ultimately led to the Weil conjectures 150 years later.

Gauss's method involved determining a conic section in space, given one focus [the sun] and the conic's intersection with three given lines [lines of sight from the earth, which is itself moving on an ellipse, to the planet] and given the time it takes the planet to traverse the arcs determined by these lines [from which the lengths of the arcs can be calculated by Kepler's Second Law]. This problem leads to an equation of the eighth degree, of which one solution, the Earth's orbit, is known. The solution sought is then separated from the remaining six based on physical conditions. In this work Gauss used comprehensive approximation methods which he created for that purpose.

Gauss also claimed to have discovered the possibility of non-Euclidean geometries but never published it. This discovery was a major paradigm shift in mathematics, as it freed mathematicians from the mistaken belief that Euclid's axioms were the only way to make geometry consistent and non-contradictory. Research on these geometries led to, among other things, Einstein's theory of general relativity, which describes the universe as non-Euclidean. His friend Farkas Wolfgang Bolyai with whom Gauss had sworn "brotherhood and the banner of truth" as a student had tried in vain for many years to prove the parallel postulate from Euclid's other axioms of geometry. Bolyai's son, Janos Bolyai, discovered non-Euclidean geometry in 1829; his work was published in 1832. After seeing it, Gauss wrote to Farkas Bolyai: "To praise it would amount to praising myself. For the entire content of the work... coincides almost exactly with my own mations which have occupied my mind for the past thirty or thirty-five years."

Gauss's presumed method was to realize that pairwise addition of terms from opposite ends ofthelist yielded identical intermediate sums: 1 + 100= 101,2 + 99 = 101,3 + 98 = 101, and so on, for a total sum of 50 x 101 = 5050.

1.6 LET US SUM UP

Mathematics is a core subject at school level and has got number of applications in daily life. Thus the teachers are expected to develop understanding about the nature of mathematics and the factors that would enable to treat mathematics as a discipline. Being the first unit of the block, we have discussed the concept and nature of Mathematics in detail. As children validate mathematical knowledge they make use of various processes such as developing hypothesis, formulating conjectures and generalisation, proving mathematical arguments and statements, etc. These are the key processes used in mathematical reasoning and validation of mathematical knowledge. In later portion of this unit, the various processes involved in mathematical reasoning have been extensively discussed citing examples pertaining to them. The unit ends with the discussion on the ways of developing skills of creativity among children.

Unit II AIMS AND OBJECTIVES OF TEACHING MATHEMATICS

2.1 The need and significance of teaching Mathematics

2.2 Aims - Practical, social, disciplinary and cultural

2.3 Instructional Objectives

2.4 General Instructional Objectives (G.I.Os) and behavioral

2.5 Specific Learning Outcomes (S.L.Os)

2.6 Relating to the cognitive, affective and psychomotor domains based on Bloom's Taxonomy of Educational Objectives.

2.1. THE NEED AND SIGNIFICANCE OF TEACHING MATHEMATICS

Mathematics is being as the mother of all sciences. If a student wants to function effectively according to the Global changes, he/she must understand the mathematics and be able to use mathematics in their daily life both personal and professional. The need and significance of teaching mathematics are as follows:

- To enable the students to get good mathematical background with the knowledge of concepts and theories.
- Make the students to apply mathematical concepts and theorems in new situations.
- To create the ability of transfer the mathematical type of thinking and reasoning to daily life situations.
- Make the students to understand the law of nature.
- Enable the students to understand the culture and development of civilization.
- Teaching mathematics gives sufficient mathematical skills to meet the demands of daily life.
- If provides better understanding of the world.
- It provides good deal of self-reliance, self-confidence, and open-mindedness.
- It promotes the skill of problem solving among the students.
- It helps to learn the other subjects which are expecting mathematical operations, particular science.

2.2.1. AIMS

The goals of education may be usefully divided into the utilitarian, social, disciplinary and cultural goals; so also the aims of teaching mathematics. Mathematics is taught in schools because it is useful, disciplines the mind and is beautiful. Every student will have to pick up mathematical knowledge and skills since it cater to the individual, societal and national needs.

While starting to teach a particular subject it is essential to know why we are going to teach that subject. Until we have clear-cut aims of teaching a subject, we would not be able to proceed on the right track Aimlessness makes the work uninteresting and results in the wastage of time, energy and other material resources both on the part of the teacher and the taught. Therefore, we must have some definite aims of teaching a subject before starting its actual teaching.

Now the question arises what would be the aims of teaching Mathematics in our schools. The answer needs the knowledge of all the advantages that can be drawn by the teaching of Mathematics as we know that aims and values are interrelated and interdependent things. One aims at a thing because one values it or by aiming at a thing one would be able to realise its values. Therefore, aims help in the realisation of values or drawing of advantages while the knowledge of the advantages or values of a subject helps in setting the aims to get all the essential advantages.

This makes us to conclude that the knowledge of the advantages or, values of teaching Mathematics may help us a lot in setting the aims of teaching Mathematics in our schools. In other words it may help us in realising what we should expect from our Mathematics teaching.

The teaching of Mathematics should essentially help the students in acquiring essential Mathematical knowledge, skills, interests and attitudes for the following purposes.

2.2.2 Practical Aim

To enable the student, to make use of the learning in Mathematics in their day to day life. To have clear ideas of number and a comprehension of the way the number is applied to measure of all varieties, but most particularly to those physical concepts he meets with most frequently, length, volume, weight, area, temperature, speed and acceleration. Able to apply his knowledge of mathematics to a wide range of problems that continually occur in his every day life. To understand the concepts of ratio and scale drawing and read, interpret graphs diagrams and tables especially those relating to statistical evidences. Able to use correctly, accurately and with understanding the four fundamental operations of addition, substraction, multiplication and division as applied to both number measurement and check both his own and other peoples calculations by appropriate approximation.

2.2.3 Social Aim

The social goals are to make the students understand how mathematics methods namely scientific, intuitive, deductive and inventive are used to investigate, interpret and to make decisions in human affairs and also how it contributes to his understanding of natural phenomena. The scientific method is that in which one seeks to discover order, pattern and relations not only in the sets or numbers in series, quantities or measures but in the natural world as well. Teachers should arrange for activities and situations to make it possible for students to discover or rediscover the relations for themselves. The intuitive method is one by which advances are made step by step by a flash of insight, and a sudden illumination of a concept brings understanding of a difficult problem situation. The results of ones insight or intuition are linked logically and thus instruction is closely related to deductive reasoning, the other method inventive method emphasizes the need for providing opportunities to students to make their own investigations and find solutions. Besides the mathematical value of discussion, co-operation, learning the importance of organization and corporate endeavour.

2.2.4 Disciplinary Aim

To develop their intellectual powers and disciplining their minds. According to Locke, "Mathematics is a way to settle in the mind a habit of reasoning." It trains or disciplines the mind. Due to its very nature, it possesses a real disciplinary value. It is exact, true and to the point knowledge, and therefore creates a discipline in the mind. Its truths are definite and exact. The learner has to argue the correctness or incorrectness of a statement. It taught in the right sense, it develops reasoning and thinking powers more and demands less from memory. The student come to realize that thinking

makes him a successful student of all the subjects. Its study results in the development of power rather than the acquisition of knowledge and knowledge also comes as a natural consequence or by-product. Reasoning in mathematics possesses certain characteristics which are suitable for the training of the learners mind. If properly emphasized and streamlined, these characteristics are likely to develop the corresponding habits in the learner. Here ensues a discussion of those characteristics and their influence.

i. Characteristic of simplicity: There is a vast scope for simple reasoning in this subject. It teaches that definite facts are always expressed in a simple language and definite facts are always easily understandable. So if you want to be understood, you must express yourself in a definite or simple way. More over, one can easily follow a gradation going from simple to complex. The teacher advances by degrees to harder and harder portions. The procedure when practiced for a pretty long time becomes a habit.

ii. Characteristic of accuracy: Without accuracy there is no chance of progress and credit in mathematics. Accurate reasoning, thinking and judgment are essential for its study. It is in the nature of this subject that it cannot be learnt through vagueness of thought and argument. In other subjects, it may sometimes be possible for the student to hide his ignorance by beating about the bush, but such tricks never play in mathematics. Accuracy, exactness and precision compose the beauty of mathematics. The student learns the value and appreciation of accuracy and adopts it as a principle of life. He learns to influence and command others by his accuracy.

iii. Characteristic of certainty of results: There is no place for subjectivity and proposal equation in mathematics. The answer is either right or wrong. Subjectivity of difference of opinion between the teacher and the taught. The student can verify his result by reverser process. It is possible for the child to remove his difficulties by self-effort and to be sure of the removal. The success of personal effort is a source of pleasure for him. He develops faith in self-effort which is the secret of success in life. He inculcates the habit of being certain about his achievement.

iv. Characteristic of originality: Most work in mathematics demands original thinking. Reproduction and cramming of ideas of others is not very much appreciated. In other subjects, of course, ideas of others occupy a prominent place and have to be grasped by the student. Therefore he can safely depend on memory in other subjects; but without original thinking and intelligent reasoning there cannot be satisfactory progress in mathematics. When he has a new or a different mathematical problem, it is only his originality which keeps him going. The discovery or establishment of a new formula is also his original work. This practice in originality enables the child to face new problems and situations with confidence in his future career.

v. **Characteristic of similarity to the reasoning of life:** Clear and exact thinking is as important in daily life as in mathematical study. Before starting with the solution of a problem, the student has to grasp the whole meaning. Similarly in daily life, while understanding a task, one must have a firm grip on the situation. This habit of thinking will get transferred to the problems of daily life also.

vi. Characteristic of verification of results: Results can be easily verified. As already pointed out, this gives a sense of achievement, confidence and pleasure. This verification of results is also likely to inculcate the habit of self-criticism and self evaluation. After making any attempt in life, the child would like to satisfy himself about its success or failure.

2.2.5 Cultural Aim

To make them understand the contribution of Mathematics in the development of culture and civilization. As regards the cultural goals the aim is to enable the student to see the part mathematics has played in the culture of the past and continues to play in the culture of the world or today. The students will have to be encouraged to preserve, promote and transmit the mathematical knowledge further to the younger generation. They have to appreciate the roles of mathematics in our modern culture and civilization through science and technology and various cultural arts like drawing, design making, painting, poetry, music, sculpture and architecture. Hence the aim is to help the student to explore the creative fields such as art, architecture and music apart from making them aware of the strength and virtues of the culture they have inherited.

Some other aims are,

Aesthetical and Recreational Aim:

To develop their aesthetic abilities, meet their varying interests and help them in the task of utilisation of their leisure time.

Moral Aim:

To help them in imbibing essential moral virtues.

Vocational Aim:

To prepare for the future vocation or occupation.

Pre-preparational Aim:

To help in the study of other subjects and future learning in Mathematics.

Inter-disciplinary Aim:

To give them insight to recognise relationships between different branches and topics of Mathematics.

Self-learning Aim:

To help them in becoming self-dependent for mastering new topics and problems of Mathematics.

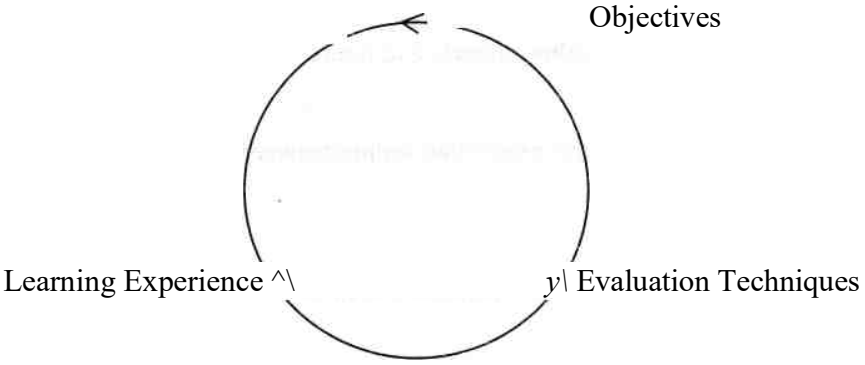
If the instructional process is to be effective, all three activities must be oriented to certain common objectives. Objectives should be started for each course, unit and topic. They are the mental skills that pupils should develop as a result of teaching. Objectives direct the pupils as to what he/she is expected to do, what should be the minimum level of acceptance for his/her performance and under what conditions it will be achieved.

2. 3 Instructional Objectives

- Define what a student be able to do after a period of instruction.
- Help the teacher plan appropriate learning experiences.
- Help the examiner plan a relevant test to actually measure these outcomes.
- Provide direction for the instructor and it clearly conveys his instructional intent to others.
- Provide the frame of reference for decisions about selection and gradation and organization of subject matter, the mode of instruction and techniques of evaluation.
- Till the students, what is expected of him after a period of instruction and these by enable him to use his study time more efficiently.

- Till the student how he should be able to use the material from the syllabus and in what ways he is expected to display his mental skills and abilities.

The interrelationship among Objectives, Learning Experiences and Evaluation Techniques.



Relationship Between Aims and Objectives

Education is given for achieving certain ends and goals. The various subjects of the school curriculum are the different means to achieve these ends. The aim is to achieve the goals or broader purposes of education. Thus by the term aims of teaching Mathematics we should mean the goals, targets or broader purposes that may be served by the teaching of Mathematics, in the general scheme of Education. Aims are like ideals. They need a long term planning. Their realisation becomes a difficult task for a subject teacher. Therefore, they are divided into some definite, functional and workable units named as objectives. Objectives of teaching Mathematics are therefore, those short term immediate goals or purposes that may be achieved within the specified classroom resources by a subject teacher. They help in bringing appropriate behavioural changes in the learning for the ultimate reahsation of the aims of teaching Mathematics.

In this way the aims of teaching Mathematics are usually broken into some specified objectives to provide definite learning experiences for bringing desirable behavioural changes. A teacher thus may have certain clear-cut well defined objectives before him at the time of teaching a particular topic.

Objectives of Teaching Mathematics at the entire school stage

The objectives of teaching Mathematics may be classified as under

a] Knowledge and understanding objectives.

b] Skill objectives.

c] Application objectives.

d] Attitude objectives.

e] Appreciation and Interest objectives.

For making the objectives unambiguous and attainable they are always expressed in the behavioural terms [testable behaviours]. What the student is expected to achieve is clearly known by the teacher while teaching a particular topic. We will try to keep this thing in view while describing the different objectives of teaching Mathematics. A.

a) Knowledge and understanding objectives:

- Through Mathematics a student acquires the knowledge and understanding of the language of

Mathematics
[in terms of symbols, formulae, figures, diagrams, technical terms and definitions].

- The various Mathematical concepts [number concept, concept of units and measurement and concept of direction etc.].
- Mathematical ideas, facts, principles, processes and relationships.
- The development of the subject and contribution of Mathematicians.
- The inter-relationship among different topics and branches of Mathematics.
- The basic nature of the subject Mathematics.

B. Skill Objectives:

Mathematics helps a student in the following ways

1. He learns and develops essential skill in the use and understanding of Mathematical language.
2. He develops speed, precision, brevity, accuracy and neatness in the computation and calculation work.
3. He learns and develops the technique of problem solving.
4. He develops the ability to estimate and check results.
5. He develops the ability to perform calculations orally or mentally.
6. He develops the ability to think correctly, to draw inferences and to generalize.
7. He develops the ability to use Mathematical apparatuses and tools skillfully.
8. He develops the essential skill in drawing geometrical figure.
9. He develops essential skill in surveying, or measuring and weighing processes.
10. He develops essential skill in drawing, reading and interpretation of graphs and statistical tables.
11. He develops skill in the use of Mathematical tables and ready reckoners.

C. Application Objectives

Mathematics helps a student in applying above knowledge and skills in the following ways

1. He is able to solve the problems of Mathematics independently.
2. He makes use of mathematical concepts and processes in everyday life.
3. He develops ability to analyse, to draw inferences and generalise the collected evidences and data.
4. He develops the ability to make use of Mathematics learning in the learning of other subjects and equips himself for higher mathematical studies.
5. He can think and express precisely, exactly and systematically by making proper use of Mathematical language.

D. Attitude Objectives

Mathematics helps in the development of correct attitudes which may be stated as below:

1. The student tries to analyse the problem.
2. He develops the habit of systematic thinking and objective reasoning.
3. He develops heuristic attitude and tries to discover the facts or solve the problems with his own independent efforts.
4. He tries to collect enough valid evidences for drawing inference, conclusion or generalization.

5. He recognizes the adequacy or inadequacy of given data in relation to the problem.
6. He tries to verify his results.
7. He understands and appreciates logical, critical and independent thinking in others.
8. He tries to express his opinions precisely, systematically and logically without any biases and prejudices.
9. He develops personal qualities *e.g.* regularity, punctuality, honesty, neatness and truthfulness.
10. He de velops proper self-confidence for solving the mathematical problems.
11. He develops mathematical perspective and outlook for observing the realm of nature and social world.
12. He shows originality and creativity.

E. Appreciation and interest Objectives:

The student is helped in the acquisition of appreciations and interests in the following way:

1. He appreciates the role of Mathematics in everyday life.
2. He appreciates the role of Mathematics in understanding his environment.
3. He appreciates Mathematics as the science of all sciences and art of all arts.
4. He appreciates the contribution of Mathematics in the development of culture and civilization.
5. He appreciates the aesthetic value of Mathematics by observing symmetry, similarity, order and arrangement in Mathematical facts, principles and processes.
6. He appreciates the contribution of Mathematicians in the development of subject and civilization.
7. He appreciates the recreational value of the subject Mathematics and learns to utilise his leisure time properly.
8. He appreciates the vocational value of the subject Mathematics.
9. He appreciates the role of Mathematical language, graphs and tables in giving precision, exactness and accuracy to the expression.
10. He appreciates the power of computation.
11. He develops interest for the learning of the subject Mathematics.
12. He feels relaxed and entertained by Mathematical recreations and amusements.
13. He takes active interest in the activities of Mathematics club.
14. He takes interest in independent library reading, working on Mathematical projects and doing practical work in Mathematics laboratory.

Objectives of Teaching Mathematics at sub-stages-Elementary and Secondary

What we want to achieve through Mathematic teaching at the entire school stages is achieved slowly and slowly through the well defined objectives, planned learning experiences and methods at the different stages.

In the 10 + 2 pattern of educational structure these stages are as follows

1. Primary or Elementary Stage [first four or five years].
2. Secondary Stage [from 5th or 6th year to 10th year of schooling].

Since it is still undecided whether the last two years should be spent in the school or college we would

like to think for the objectives of teaching Mathematics at the elementary and secondary stage only.

Objectives of Teaching Mathematics at the Secondary Stage

Objectives of teaching Mathematics at this stage are more or less the same as listed earlier for the entire school stage. However, the objectives of teaching Mathematics at the elementary stage [being elementary in nature] may be specified as follows :-

Objectives of Teaching Mathematics at the Elementary Stage

Objectives of teaching Mathematics at this stage may be specified in terms of the following expected learning outcomes of student behaviour:

A. Knowledge and Understanding Objectives:

The student develops knowledge and understanding for the following:

1. Concepts like number concept, concept of units of measurement, concept of size and shape, concept of fractions, concept of direction and distance, concept of grouping and sub grouping.
2. Mathematical facts and processes like the place values of numbers, the meaning and significance of zero, the four fundamental operations, L.C.M. and H.C.F., Percentage, Unitary Method, Simple Interest, Profit and Loss and Mensuration.
3. Arithmetical terms and symbols like digits and numbers, symbols for fundamental operations, fractions, percentage etc.
4. Relationship between different topics of arithmetic.

B. Skill Objectives:

The students develops the following skills :-

1. He develops ability in reading, writing and counting of numbers.
2. He develops skill in four fundamental operations dealing with integral numbers and fractions.
3. He develops a reasonable speed, accuracy and neatness in the computation of oral as well as written work in Mathematics.
4. He develops the technique of solving problems involving elementary Mathematical processes and simple calculations
5. He develops skill in the use of Multiplication Tables.
6. He develops proficiency in making quantitative estimate of size and distance.

C. Application Objectives:

The student is able to apply the above knowledge and skills as follows:

1. He is able to solve elementary mathematical problems independently [orally as well as in writing].
2. He makes use of elementary mathematical concepts and processes in every day life.

D. Attitude Objectives:

1. The student develops proper self-confidence for solving elementary mathematical problems.
2. For the solution of a problem he tries to read it carefully, analyses, collects all the known evidences and then draw proper inferences.
3. He develops habit of regularity, neatness, truthfulness and honesty.

E. Appreciation and Interest Objectives:

1. He develops interest for the learning of the subject Mathematics.
2. He appreciates the contribution of Mathematicians and gets inspiration from their work.
3. He appreciates the power of computational skills.
4. He appreciates and takes interest in using his learning of Mathematics in solving daily life problems.
5. He appreciates the recreational value of the subject Mathematics and learns to utilise his leisure time properly.

2.4 & 2.5 THE GENERAL INSTRUCTIONAL OBJECTIVES [GIO'S] AND SPECIFIC INSTRUCTIONAL OBJECTIVES[SIO'S]

An objective of instruction is an instructional objective, a statement of what the learner is likely to be after successfully completing the specified learning task.

General instructional objectives are stated using terms as knows, understands, applies, appreciates, develops etc. the student understands the base two system of numbers [This is not an observable behaviour as understanding by the student cannot be observed by the teacher]. The GIO begins with verbs such as knows, develops, appreciates etc which are not action verbs.

Specific instructional objectives[SIO's] are the learning outcomes or terminal student behaviour which are observable; they are action verbs such as defines, states, compares, distinguishes, identifies, illustrates, classifies, computes, converts, checks, derives, extracts, groups, solves, tabulates, constructs, verifies, interprets, discriminates etc. The student converts a number in base ten into number in base two system [This is an observable behaviour because converting from one form to the other can be observed by the teacher]. The SIO are the expected learning outcomes after the instruction. The SIO begins with action verbs such as recalls, compares, identifies measures etc. which indicate terminal behaviours that are definite and observable.

The GIO and SIO when stated clearly give direction for action to bring about the desired changes in the students behaviour enabling them to attain the goals or aims.

Writing Objectives in Behavioural Terms

The major weakness about the taxonomies of objectives given above lies in the fact that they do not state objectives in terms of terminal behaviour i.e. what the learner should be able to do at the end of teaching. Specification of objectives in a task of teaching and learning may prove more effective and purposeful if they are translated into behavioural language.

The structure of the educational or instructional objectives mainly consists of two parts, namely,

[i] the modification part and [ii] the content part.

The modification part represents the behavioural changes that are designed to occur in the behaviour of the learner through the related instruction or learning experiences.

The content part refers to the syllabus in particular and to the curriculum in general to be covered by the related instruction.

Therefore, the writing of an objective in behavioural terms is always done in related to the following three things

i. The nature of the objective *i.e.* knowledge, application etc.

ii. The area or domain of the behaviour *i.e.* cognitive, affective etc.

iii. The specific content areas in which behavioural changes are planned to be brought about *i.e.* Fundamental rights, Means of irrigation, Sources of heat, etc.

A list of Associated Action Verbs for the Cognitive Domain Objectives [Based on Bloom's Taxonomy]

1. **Knowledge:** Define, List, Label, Measure, Name, Recall, Recognise, Reproduce, Select, State, Write, Underline, etc.
2. **Comprehension:** Change, Classify, Distinguish, Explain, Formulate, Identify, illustrate, Indicate, Interpret, Justify, Judge, Name, Represent, Select, Summarize, Transform, Translate, etc.
3. **Application:** Assess, Change, Choose, Conduct, Construct, Compute, Demonstrate, Discover, Explain, Establish, Find, Generate, Illustrate, Modify, Predict, Perform, Select, Solve, Use, etc.
4. **Analysis:** Analyse, Associate, Compare, Conclude Contrast, Criticize, Differentiate, Identify, Justify, Point out, Resolve, Select, Separate, etc.
5. **Synthesis:** Argue, Conclude, Combine, Derive, Discuss, Generalize, Integrate,Organise, Precise, Prove, Relate, Restate, Select, Summarize, Synthesize, etc.
6. **Evaluation:** Associate, Choose, Compare, Criticize, Conclude, Defend, Determine, Evaluate, Judge, Identify, Recognize, Relate, Select, Summarize, Support, Verify, etc.

A list of Action Verbs for Affective domainObjectives [Based on Bloom's Taxonomy]

1. **Receiving:** Ask, Accept, Attend, Beware, Catch, Discover, Experiment, Identify, Favour, Follow, Observe, Prefer, Perceive, Receive, Select, etc.
2. **Responding:** Answer, Assist, Complete, Derive, Discuss, Develop, Help, List, Label, Name, Obey, Present, Practise, Record, Select, State, Write, etc.
3. **Valuing:** Accept, Attain, Complete, Choose, Decide, Demonstrate, Discriminate, Develop, Increase, Indicate, Influence, Participate, Prefer, Recognise, etc.
4. **Organising:** Add, Associate, Change, Compare, Complete, Coordinate, Correlate, Determine, Find, Form, Generalize, Integrate, Judge, Project, Prepare, Relate, Select, synthesize, Organise, etc.
5. **Characterising:** Accept, Change, Characterize, Decide, Discriminate, Demonstrate, Develop, Experiment, Face, Identify, Judge, Prove, Revise, Serve, Solve, Verify, etc.

A list of Action Verbs for Psychomotor domain Objectives

1. **Imitation:** Chooses, describes, detects, differentiates, distinguishes, identifies, isolates, relates, selects, separates
2. **Manipulation:** begins, displays explains, moves, proceeds, reacts, responds, shows, starts, volunteers
3. **Precision:** measures, sketches, mends, mixes
4. **Articulation:** Assembles, builds, displays, organizes
5. **Naturalization:** composes, constructs, originates

Instructional Objectives and their relationships to General Aims and Objectives of Teaching Mathematics

At the time of imparting instruction *i. e.* teaching-learning of a particular lesson, unit or subunit of the subject mathematics, teacher has to place before him some definite and very specific objectives for being attained within a specified classroom period and resources in hand. Through these so specific classroom teaching-learning objectives, known as instructional objectives, a teacher tries to bring desired changes in the behavior of his pupils. In this way, the term instructional objectives in relation to the teaching of mathematics may be defined as a group of statements formulated by the teacher for describing what the pupils are expected to do or will be able to do once the process of classroom Instruction is over. In fact instructional outcomes is the teaching-learning product in the form of behavioral changes in the pupils that a teacher expects as a result of his instruction related with a particular lesson, unit or sub-unit of the subject. Instructional objectives are thus nothing but descriptions of the pupil's terminal behavior expected out of the on going classroom instruction.In comparison to general aims and objectives of teaching mathematics, instructional objectives are quite narrow and specific. They are definite, tangible, precise and functional. They are predetermined and are always formulated in such a way that their attainment becomes quite practicable through the usual classroom teaching within the stipulated period of fixed duration. They are the desired learning or teaching outcomes and are always stated in terms of expected pupil's behaviour or desired behavioural changes. They are, therefore, may be termed as teaching-learning objective or behavioural objectives. The main purpose of these objectives is to provide statements of skills, concepts or the behaviour, learners are expected to demonstrate after going through particular instruction. Objectives of teaching mathematics falls midway between goals or aims of teaching mathematics and instructional objectives. They are more specific and definite than the general aims or goals but less specific and much wider than the classroom instructional objectives. Their attainment is quite possible within the educational structure and means.

Classroom instructional objectives may be submerged in the reservoir of general objectives of teaching mathematics at a particular school stage which in turn are further submerged in the ocean of general aims and goals of teaching mathematics.

2.6 BLOOM'S TAXONOMY OF INSTRUCTIONAL OBJECTIVES

"Taxonomy" means a system of classification and in this sense a taxonomy like Bloom's Taxonomy presents a system of classification of the objectives in the similar way as Dewey's Decimal system tends to classify a number of books in a library.

The taxonomy, of educational and instructional objectives has been worked out on the assumption that the teaching-learning process may be conceived as an attempt to change the behaviour of the pupils with respect to some subject matter or learning experiences Behaviour is divided into three domains—Cognitive [knowing], affective [feeling] and psychomotor [doing]. The taxonomy of educational and instructional objectives have also been considered to be belonging to these three domains.

The taxonomy related to cognitive' domain has been presented by Bloom [Bloom, et. al, 1956], the second related to affective domain by Krathwohl, Bloom and Mosia [Krath Wohi,' et al, 1964] and the third related to psychomotor domain by Harrow' [Harrow, 1972] and Simpson [1966].

Let us have a brief description of these taxonomies.

Taxonomy of objectives in the Cognitive Domain

Bloom and his associates have classified the objectives related to cognitive domain into six categories arranged from the lowest to the highest level of functioning as described below.

1. Knowledge

a. Knowledge of specifics
b. Knowledge of terminology.
c. Knowledge of specific facts.
d. Knowledge of ways and means of dealing with specifics.
e. Knowledge of conventions.
f. Knowledge of trends and sequences.
g. Knowledge of classifications and categories.
h. Knowledge of criteria.
i. Knowledge of methodology.
j'. Knowledge of universals and abstractions in a field.
k. Know 1 edge of principles and generalizations.
l. Knowledge of theories and structures.

2. Comprehension

a] Translation
b] Interpretation
c] Extrapolation

3. **Application**

4. Analysis

a] Analysis of elements
b] Analysis of relationships
c] Analysis of organisational principles

5. Synthesis

[a] Production of unique communication
[b] Production of a plan or a proposed set of operations
[c] Derivation of a set of abstract relations

6. Evaluation

a] Judgement in terms of internal evidence
b] Judgement in terms of external criteria.

Let us try to elaborate the above taxonomy of objectives of cognitive domain given by Bloom

1. Knowledge. It represents the lowest level of the objectives belonging to the cognitive domain and primarily aims for the acquisition of the knowledge concerning.

[i] Specific facts, terminology, methods and processes and

[ii] Generalized principles, theories and structures.

The knowledge objectives mainly call for the recall and recognition level of one's memory and therefore their evaluation is primarily made through a simple recall or multiple choice type questions.

2. Comprehension. Comprehension is based upon the knowledge. If there is no knowledge, there will be no comprehension. On the ladder of the acquisition of cognitive abilities its level is little higher than the knowledge. Specifically, it means the basic understanding of the facts, ideas, methods, processes, principles or theories, etc. As a result, what is communicated to a learner, he may

i. translate or summarize the communicated knowledge in his own words

ii. interpret *i.e.* cite examples, discriminate, classify, verify or generalize and

iii. extrapolate *i.e.* estimate or understand the use of knowledge and extend it to other subjects and fields.

3. **Application**. The knowledge is useful only when it is possible to make it employed. The application of an idea, principle or theory may be made possible only .when it is grasped and understood properly, Therefore, the category of application automatically involves both the earlier categories *i.e.* knowledge and comprehension. Under this objective the learner is required to acquire the ability to make use of the abstract or generalized ideas, principles in the particular and concrete situations.

4. **Analysis**. Analysis refers to an understanding at higher level. It is a complex cognitive process that involves knowledge, comprehension as well as application of an idea, fact, principle or theory. Through the realization of these objectives the learner is expected to acquire the necessary skill in drawing inferences, discriminating, making choices and selection and separating apart the different components or elements of a concept, object or principle.

5. **Synthesis**. The objectives belonging to this category aim to help the learner to acquire necessary ability to combine the different elements or components of an idea, object, concept, or principle as to produce an integrated picture [i.e] figure of wholeness. As a result he may be expected to propagate or present a theory or principle by combining different approaches, ideas or view points. He may arrive at something new or originate some novel thing or idea after synthesizing all what is known to him earlier. In this war, it calls for the creativity aspect of the cognitive abilities and therefore may be considered definitely a higher level of learning involving knowledge, comprehension, application as well as analysis.

6. **Evaluation**. This category of objectives aims to develop in the learner the ability to make proper value judgement about what has been acquired by him in the form of knowledge, understanding, application, analysis and synthesis. It involves all the five categories described earlier. As a result the learner is expected to take proper decision about the quantitative and qualitative value of a particular idea, object, principle or theory. He may arrive at an appropriate decision about the matter and methods by making use of all the cognitive abilities acquired through the earlier categories of cognitive objectives.

Taxonomy of objectives in the Affective Domain

Taxonomy of Objectives in the Affective Domain arranged from lowest to highest level of functioning.

1. **Receiving** [Attending]
   - [a] Awareness
   - [b] Willingness to receive
   - [c] Controlled or selected attention
2. **Responding**
   - [a] Acquiescence in responding
   - [b] Willingness to respond
3. **Valuing**
   - [a] Acceptance of a value
   - [b] Preference for a value
   - [c] Commitment
4. **Organization**
   - [a] Conceptualization of a
   - [b] Organisation of a value system
5. **Characterization by a value or value Complex**
   - [a] Generalized set
   - [b] Characterisation

Let us try to elaborate further the above classification for its clarity and understanding as below.

1. Receiving. It represents the initial category for the objectives belonging to affective domain. For the inculcation of certain interests, attitudes, values or ideas it is essential that learner should be made to receive or attend the desired ideas, events or objects. This category points out towards this necessity and takes into consideration three types of following sequential activities :-

[i] Firstly, the learner is sensitized or made aware about the existence of certain stimuli.

[ii] Then the desired intention or willingness for receiving or attending the stimuli is created in the learner.

[iii] Lastly, the efforts are made for the control of the attention of the learner. He may be trained to pay selective attention and sustain it for a desired period.

2. Responding. It represents the second level of the objectives for the categories belonging to affective domain. Once a learner receives or attends to a particular idea, event

or thing he must be made to respond to it as actively as possible. The responses here do not confine itself in just paying attention or arousal of a simple intention or desire of getting a thing, as in the first category of receiving but manifest, themselves in the active behaviour like obeying, answering, reading, discussing, recording, writing and reacting to a stimulus, etc.

3. Valuing. When one attends as well as responds to a particular thing, idea or event he is naturally drifted towards taking value judgement about that thing, idea or event. Therefore, this category of valuing depends upon both the former categories *i. e.* receiving and responding. Here the learner is

expected to imbibe a definite value pattern towards different ideas, events, and objects. In practice the objectives belonging to this category are usually concerned with the development of typical value patterns, attitudes, etc.

4. Organising. This category of objectives concern with the construction of relatively enduring value structure in the learner by organising and synthesizing the different value patterns imbibed by him from time to time. Ultimately this category of objective leads the learner to form a set value structure or philosophy of life.

5. Characterizing by a value or value complex. It is the highest level category of the objectives belonging to the affective domain. Up to this stage, the learner is able to imbibe all the essential affective behaviour *i.e.* various interests, attitudes, values, value complex or value patterns, a permanent set value structure and therefore, all the earlier categories are automatically involved in the objectives of this category. At this stage, the learner is destined to imbibe typical characteristics of his individual character i.e. life style of his own- In fact it is the end point or ultimate goal of the process of education.

Taxonomy of objectives in the Psychomotor Domain

Taxonomy of Psychomotor Objectives [R.H. Dave] *

1. Imitation

[a] Impulsion

[b] Over t repetition

2. Manipulation

[a] Following direction [bj Fixation

3. Precision

[a] Reproduction

[b] Control

4. Articulation

[a] Sequence

[b] Harmony

5. Naturalization

[a] Automatism

[b] Internalization

Let us have a necessary explanation for the above steps

Imitation For the learning of a psychomotor activity i.e. drawing or surveying skill in mathematics, the task begins with the imitation of observed acts. The child observes the demonstrated behaviour related to drawing of a line, angle or circle etc. He feels an inner push or an impulse [by having an inner rehearsal of the psychomotor activities] to imitate action It is followed by the overt repetition [imitation] of the demonstrated behaviour.

Manipulation. This second category of psychomotor objectives emphasizes manipulation on the part of the learner for the acquisition of skills by following directions, performing selected action and fixation of performance through necessary practice.

Precision. In the third category of psychomotor objectives learner is able to perform skilled acts or motor activities with a desired level of precision [accuracy, exactness and right proportion] and as such may be said to reach a higher level of refinement in reproducing a given act or skilled task.

Articulation. It is the fourth category in the hierarchy of learning the psychomotor activities or skills. At this stage learner becomes capable of coordinating a series of acts by establishing appropriate sequence and accomplishing harmony or internal consistency among different acts.

Naturalization. It is the highest stage reached in terms of the development or proficiency acquired in the learning of a skill or psychomotor act. One can now perform a single act or a series of articulated acts with a greater refinement, ease and convenience as automatic and naturally as possible.

Unit III LESSON PLANNING AND ITS USES

3.1 Micro teaching

- Origin, need, procedure, cycle of operation and uses
- Skill emphasis
- Explaining, questioning
- Probing and Fluency in questioning, using black board, reinforcement, Stimulus variation, introduction, Closure

3.2 Link Lesson

3.3 Macro teaching

– Lesson plan, Unit plan & Year plan

– Herbartian steps

- Format of a typical lesson plan

– G.I.O‘s & S.I.O‘s, teaching aids

– Motivation, presentation, application, recapitulation and assignment

3.1 MICRO TEACHING: ORIGIN

Micro teaching was first introduced at the Stanford University, USA in 1963. The Stanford Teacher Education staff members sought to identify, isolate and build training programmes for critical teaching skills. Micro teaching, has since then been refined and applied not only in teacher training but also in business, Nursing and army research in India and other developing countries has shown that conventional micro teaching methods help to improve teaching competence.

Micro teaching is now considered not only as a constructive teacher training teaching technique but also as a versatile research tool which dramatically theologises investing of certain teaching skills and learning variables. Teaching constitutes a number of verbal; and non-verbal acts. A set of related behaviours or teaching acts aiming at specific objectives and performed with an intention to facilitate pupils learning can be called a teaching skills. All these technical skills which go to make good teaching can be defined, observed, measured, controlled by means of practice. Micro-teaching concentrates on specific teaching behaviours and provides opportunity for practicing teaching under controlled conditions. Micro teaching is a method of teacher training and not a method of teaching in class room. It simplifies the complex teaching process so that the student teacher can cope with it.

Need for Micro Teaching:

The heart of any educational process is teaching which includes training, instruction and development of cognitive processes and abilities. Experts have commented that the quality of a nation is judged by the quality of its education, which is in turn, is decided by the type of teachers it has. The teacher in a classroom uses several techniques and procedures too bring about effective learning on the part of his students. These activities include introducing, demonstrating, explaining or questioning and the teacher could also use nonverbal behaviours such as smiling, gesturing and nodding. These activities form what are called teaching skills. In order to achieve objectives in all three domains the teacher has to acquire all teaching skills and use them appropriately. Hence it is necessary that a student teacher is

introduced to a quite range of teaching skills. Micro teaching allows the student teacher to practice any skill independently and integrate it with other skills in familiar environment.

Meaning and Definitions of Micro teaching:

The teacher education programme is made up of two parts: the theoretical course and the practical course with the main focus on practice teaching. The student teachers are provided with a relatively strong theoretical base and are exposed to a couple of demonstration and observation lessons: then they have to actually practical or put into action what they have seen and heard, in a real classroom situation. Here he learns the art and craft of teaching by trial and error. To rectify this micro teaching approach was introduced. Micro teaching is a procedure in which a student teacher practices teaching with a reduced number of pupils in a reduced period of time with emphasis on a narrow and specific teaching skill. Microteaching is a scaled down teaching act, scaled down in terms of class size and class time.

D.W. Allen defines "Micro teaching is a scaled down teaching encounter in class size and time".

Allen and Eve defined micro teaching as a system of controlled practice that makes it possible to concentrate on specific teaching skills and to practise teaching under controlled conditions.

David B.Young defines "Micro teaching as a device which provides the novice and experienced teacher alike, new opportunities to improve teaching".

Clift and others defined it as a teacher training procedure which reduces the teaching situation to simpler ad more controlled encounter by limiting the practice teaching to a specific skill and reducing the teaching time and class size.

Characteristic features of Micro teaching:

- Micro teaching is an experiment in the field of teacher education which has been incorporated in the practice teaching schedule.
- It is a student teacher skill training technique and not a teaching technique or method like lecture - demonstration or inductive-deductive methods.
- Micro teaching is micro or minature in the sense that it scales down the complexities or real teaching with provisions of
    - Practicing one skill at a time
    - Reducing the class size to 5-10 pupils.
    - Reducing the duration of lesson to 5-10 minutes.
    - Limiting the content to a single concept.
- Micro teaching advocates the choice and practice of one skill at a time.
- Feed back is provided immediately after the completion of the lesson.
- It is an analytic approach to training.
- Micro teaching is a highly individualized training device permitting imposition of a high degree of control in practicing a particular skill.

Components of Micro Teaching

A. T *u*

v Teacher trainee

- Students [5-10]
- Observers [2]
- Supervisor [1]

Micro teaching procedure

In a micro teaching procedure the student teacher is involved in a scaled down teaching situation-in terms of class size, class time and teaching tasks. The tasks may include practicing and mastering of a specific teaching skill such as explaining, questioning, introducing, mastering of specific teaching strategies, flexibility; use of instructional materials and class room management.

The short lesson are taken by the student teachers. The pupiis who attend the lesson are asked to fill in the rating questionnaires and evaluate the specific aspects of the lesson. The supervisor also records and suggests remedial measures. If asked to reteach, the student teacher replans his lesson and immediately reteaches the lesson to the group of peer members which is observed, recorded and feedback provided. This cycle continues until he masters the specific skill. The micro teaching cycle is represented below.

Micro Teaching Cycle of operation

Steps of **Micro-teaching**

The Micro-teaching programme involves the following steps :

Step I

Particular skill to be practised is explained to the teacher trainees in terms of the purpose and components of the skill with suitable examples.

Step II

The teacher trainer gives the demonstration of the skill in Micro-teaching in simulated conditions to the teacher trainees.

Step III

The teacher trainee plans a short lesson plan on the basis of the demonstrated skill for his/her practice.

Step IV

The teacher trainee teaches the lesson to a small group of pupils. His/Her lesson is supervised by the supervisor and peers.

Step V

On the basis of the observation of a lesson, the supervisor gives feedback to the teacher trainee. The supervisor reinforces the instances of effective use of the skill and draws attention of the teacher trainee to the points where he could not do well.

Step VI

In the light of the feed-back given by the supervisor, the teacher trainee replans the lesson plan in order to use the skill in more effective manner in the second trial.

Step VII

The revised lesson is taught to another comparable group of pupils.

Step VIII

The supervisor observes the re-teach lesson and gives re-feed back to the teacher trainee with convincing arguments and reasons.

Step IX

The 'teach - re-teach' cycle may be repeated several times till adequate mastery level is achieved.

Uses of Micro Teaching

- It is an effective device for modifying the behaviour of teachers under training.
- It is highly individualized type of teacher training.
- It is useful for pre-service and in-service teacher training. The teachers can improve their competency of teaching.
- Feed back being quick, there is scope for early remedy of drawbacks and hence over all improvement in teaching sis possible.
- It provides a lot of scope for research work especially of experimental type.
- It helps in developing useful type of curriculum.
- Usually class room teaching is a complex and complicated type of activity, but micro teaching simplifies it so as to make it suitable for the beginner teachers.
- It helps in acquiring various types of skills which ultimately forms the basis of successful teaching.
- It develops a lot of confidence in the teachers.
- It helps in sorting out problems related to class room teaching, proper solutions can be thought of.
- Close supervision is possible.
- The objectives of micro lesson are clearly given in behavioural terms.
- It caters to the individual differences as opportunities are provided for re-planning and re-teaching till a skill is mastered.
- It reduces time and energy as there is no room for trial and error.
- Modification of teacher behaviour and learning of specific tasks are the main outcomes of micro teaching.

Merits of Micro-teaching

- It helps to develop and master important teaching skills.
- It helps to accomplish specific teacher competencies.
- It caters the need of individual differences in the teacher training.
- It is more effective in modifying teacher behavior.
- It is an individualized training technique.
- It employs real teaching situation for developing skills.

- It reduces the complexity of teaching process as it is a scaled down teaching.
- It helps to get deeper knowledge regarding the art of teaching.

Phases of Micro-teaching

Micro-teaching has the following three phases.

Phase - I : Pre-active phase

Phase - II : Interactive phase

Phase - III : Post-active phase

Phase - I : Pre-active phase

Phase I emphasises on the understanding of the teaching skill that is to be leant by the teacher trainee. It envisages the followings,

1] Orientation to micro-teaching

2] Discussion of teaching skills with their components and teaching behaviours.

3] Presentation of model demonstration lesson by the teacher educator.

4] Observation of the model lesson and criticism by the teacher trainees.

Phase - II : Interactive phase

Objective of interactive phase is to enable the teacher - trainee to practice the teaching skill following the microteaching cycle like,

1] Preparation of micro-lesson plan for the selected teaching skill.

2] Creating microteaching setting

3] Practice of teaching skill

4] Feedback 5] Re-planning 6] Re-teaching

7] Repetition of the microteaching cycle.

Phase - III : Post-active phase

Objective of Post-active phase is to enable the teacher - trainee to irrigate the teaching skill in real or normal classroom situation. Integration may be defined as the process of selection, organization and utilization of different teaching skills to form an effective pattern for realising instructional objectives in a teaching - learning process.

Principles of Micro-teaching

- Micro-teaching is based on the premise that teaching can be analysed into various component behaviours called teaching skills.
- The teaching skills can be defined, practised, observed, controlled, measured and evaluated.
- Micro-teaching technique seems to be based on skinner's operant conditioning i.e. reinforcing an operant response increases the possibility its recurrence of a response. This principle is fundamental to the feedback session.
- Skinner's theory of shaping in acquiring new patterns of behaviour seems to have been, applied to 'teach-feedback - reteach' pattern in the micro-teaching.

- The steps involved in behaviour modification are,

Stating the behaviour in operational terms o Fixing criteria for operational terms. o Pre-treatment stage involving measuring entry behaviour. o Giving actual treatment for behaviour modification. o Obtaining post-treatment measures.

3.1.1 MICRO-TEACHING SKILLS (SKILL EMPHASIS)

Micro-teaching skills according to 'Allen and Ryan' are as follows:

1. Skill stimulus variation
2. Skill of set Induction
3. Skill of closure
4. Teacher silence and non-verbal cues
5. Skill of Reinforcing pupil participation
6. Skill of fluency in questioning
7. Skill of Probing questioning Skill of Higher questions Skill of Divergent questions
10. Skill of Recognizing and attending behaviour
11. Skill of Illustrating and use of examples
12. Skill of Lecturing
13. Skill of planned repetition
14. Skill of completeness of communication.

Micro-teaching skills according tot he P.K. Parsi are as follows:

1. Skill of Writing Instructional Objectives
2. Skill of Introducing a lesson
3. Skill of Fluency in questioning
4. Skill of Probing questioning
5. Skill of Explaining
6. Skill of Illustrating with examples
7. Skill of Stimulus variation
8. Skill of Silence and non-verbal cues
9. Skill of Reinforcement
10. Skill of Increasing Pupil participation
11. Skill of Using black board
12. Skill of Achieving Closure
13. Skill of recognizing attending behaviour.

Micro-teaching skills identified by the NCERT in its publication of 'Core Teaching Skills' are as follows,

1. Skill of Writing Instructional Objectives.
2. Skill of Organizing the content

3. Skill of Creating set for Introducing the lesson
4. Skill of Introducing a lesson
5. Skill of Structuring classroom questions
6. Skill of Question delivery and its distribution.
7. Skill of Response Management
8. Skill of Explaining
9. Skill of Illustrating with examples
10. Skill of Using teaching aids
11. Skill of Stimulus variation
12. Skill of pacing o the lesson
13. Skill of Promoting pupil participation
14. Skill of use of Blackboard
15. Skill of Achieving closure of the lesson
16. Skill of Giving Assignments
17. Skill of Evaluating the pupil's progress
18. Skill of Diagnosing pupil learning difficulties and taken remedial measures
19. Skill of Management of the class.

Skill of Explaining

A teacher is said to be explaining when he is describing How, Why, What concepts, events, actions and etc., It can be described as an activity to bring about an understanding in the learner about a concept, principle etc.

Explaining bridges the gap in understanding the new knowledge by relating it to the past experience. Explaining depends upon the type of the past experience, the type of the new knowledge and the type of the relationship between them.

Explanations can be made more effective by using simple and clear language for clarity, examples and illustration materials for better understanding and appropriate link words for relating the concepts.

Components of Skill of Explaining

- Beginning Statement
- Link words
- Concluding Statement
- Questions to test pupils understanding
- Correct answers getting from the questions

Undesirable behaviours in the skills of Explaining

1) Making Irrelevant Statements:

Statements which are not related sand do not contribute to the understanding of the concept being explanied are irrelevant for explanation.

2) Lack of continuity:

Lack of continuity in terms of logical sequence, relationship with previous statement, references to earlier experiences and so on have to be avoided.

4.2.1.3. Model micro lesson plan [Skill of Explaining]

Name of the teacher - trainee : xxxx

Skill : Explanation

Topic : Mensuration

Teachers : In the last class we have desired the formula to find the area of a right -angled triangle. What is the formula? [beginning statement]

Student : 1/2 ab [Where a, b are the sides]

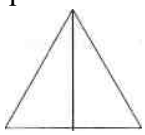
Teacher

B C

Students

What kind of triangle is ABC? Why? [Beginning Statement] ABC is a equilateral triangle because all the sides are equal. A

Teacher



B D C

In AABC, BC is the base, AD is perpendicular to BC.

Therefore, [Link Word] AD is the attitude.

Teacher

What king of triangles are AABD and AACD? Why?

[Questions to test pupils understanding]

AADB and AACD are right angled triangle because ZADB=ZADC = 90°

Student

[Correct answers getting from the questions asked]

Teacher : So [Link word] in a triangle, among the three angles any one has 90°, it is said to be a right - angle triangle [Concluding statement].

Observations sheet

Components	**Very good**	**Good**	**Average**	**Poor**	**Very poor**
Beginning Statement					
Link words					
Concluding Statement					
Questions to test pupils understanding					
Correct answers getting from the					

Skill of Questioning

Questions are the most important tool of thinking, reasoning, learning and teaching. Questions are used at every stage of teaching.

i.e. pre-active, interactive and post-active stages.

Objectives of Questioning

1. Finding out previous knowledge
2. Revising the topic
3. Stimulating though process
4. Encouraging discussion
5. Getting students participation

Questioning Approaches

1] Irrigating questions:

A list of questions is put to a particular student or a group of student. After which it is rotated in the class from one student to another.

2] **Delivery**

In this type of approach all participants have to consider the questions. Students are given time to think about the answers after teaches calls individual students and asks them to give answers.

3] **Acknowledging the answers**

Sometimes teacher put questions to acknowledge the answer of the previous question and reinforce their positive response.

Types of questions

1] Open questions

Questions for which there is no a single definition answer.

2] Closed questions

Questions which have only one exact correct answer.

3] Rhetorical questions

Questions for which there are no easy answers. Those are highes order thinking questions.

Components of skill of Questioning:

1]Structure

2]Process

3] Product

Skill of probing and Fluency in questioning

Probing questions are those which help the students to think in depth about the various aspects of the problem. By asking such questions again, the teacher makes the public more thoughtful. Teacher enables the students to understand the subject deeply.

Components of skill of probing and fluency in questioning

1. Prompting
2. Seeking further information
3. Refocusing
4. Redirecting Questions
5. Increasing Critical Awareness

Undesirable behaviours in the skill of Probing and Fluency in questioning

- Very often or often questions and answer could make the students inattentive.
- Reframing the questions could result in confusion.

- Suggestive questions should be avoided.
- Questions requiring the student to respond by 'yes or no' are leading questions and do not stimulate the students to think.

Observation sheet

Components	Very good!	Good	Average	Poor	Very poor

Skill of using Black Board

Blackboard is an important visual aid used by teachers for effective teaching. A mathematics teacher makes extensive use of blackboard for daily classroom teaching for working out problems, deriving formulae, proving theorems, drawing figures, constructing geometrical figure and etc.

Purpose of Black board

1] Effective visual aid
2] Provides clarity in understanding concepts 3] Draws attention of students at relevant points 4] Presents holistic picture of the content.

Components of skill of using Black board

1] Legibility of hand writing
2] Neatness
3] Appropriateness
4] Orgnization

Observation sheet

Components	Very good	Good	Average	Poor	Very **poor**
Legjbility of hand writing					
Neatness					
Appropriateness					
Organization					

Skill of reinforcement

Reinforcement is strengthening the connection between a stimulus and a response.

Types of reinforcements

i] Positive reinforcements ii] Negative reinforcements.

i] Positive reinforcements •

Positive reinforcements provide pleasant experience or a feeling of satisfaction which contributes towards strengthening of desirable responses.

ii] Negative reinforcements.

Negative reinforcements result in unpleasant experiences, which help in weakening the occurrences of undesirable responses.

The skill of reinforcement increases the student involvement towards learning. The skill is used, when the teacher reinforces correct responses with a smile, when the teacher praises a good response or encouraged a slow learner. The use of more and more positive reinforces maximize students involvement towards learning rather than the use of negative reinforces.

Components of skill of reinforcement

1] Positive Verbal reinforcement
2] Positive non-verbal reinforcement
3] Negative Verbal reinforcement
4] Negative non-verbal reinforcement
5] Contact reinforcement
6] Proximity reinforcement
7] Activity reinforcement
8] Token reinforcement
9] Inappropriate use of reinforcement
10] Denial reinforcement

Components	Description
Positive Verbal reinforcement	Repeating students answer, Good, very good, excellent, right.
Positive non-verbal	Smiling, nodding the head, moving towards the student when correct response, mm-hmm etc.
Negative Verbal	Nonsense, Very bad, No, worst etc. ,
Negative non-verbal	Anger face reaction, Negative head movements etc
Contact reinforcement	Patting the back, hand shaking, putting hands on the students head.
Proximity reinforcement	Going nearer to the pupils and making them more involved and interesting in learning
Activity reinforcement	Giving a task, project, home work, assignment,
Token reinforcement	Awarding marks, grader, etc.
Inappropriate use of reinforcement	When the teacher does not encourage the student with respect to quality of his response. Teacher uses same type of comment for every response.
Denial reinforcement	The teacher does not give reinforcement when the situation is demanding encouragement.

Undesirable components of skill of reinforcement

1] Negative verbal reinforcement
2] Negative non- verbal reinforcement
3 Inappropriate use of reinforcement
4. Denial reinforcement

Observation Tally sheet

Components	**Tally**	**Total**
1. Desirable Components		
1] Positive verbal reinforcement		
2] Positive non-verbal reinforcement		
3] Contact reinforcement		
4] Proximity reinforcement		
5] Activity reinforcement		
6] Token reinforcement		
II. Undesirable components		
1] Negative verbal reinforcement		
2] Negative non-verbal reinforcement		
3] Inappropriate use of reinforcement		
4] Denial reinforcement	V	

Skill o stimulus variation

If the classroom environment becomes monotonous, then it put a negative impact on the teaching and learning process. It is therefore, essential to make the classroom environment challenging and interesting such that the teaching - learning process becomes lively, interesting, pleasant and a though provoking experience. This process of brining variation in the overall interactive environment of the class with the help of stimuli change is called stimulus variation.

It is very important for a teacher to secure and sustain student's attention. For this purpose, the teacher uses some pester, body movements, verbal statements, etc. All these behaviours are related to stimulus variation. The skill of stimulus variation can be defined as deliberate change in the attention drawing behaviours of the teacher in order to secure and sustain student's attention towards the lesson.

Components of the skill of stimulus variation

1] Movement
2] Gesture
3] Change in interaction style
4] Change in speech pattern
5] Oral - visual switching
6] Pupil activity
7] Focusing

Components	Description
]] Movement	Teacher should not be static while teaching. He/she should more around the teacher's table. So that both the teacher and students be active.

2] Gusture	The Body language and the facial expression of the teacher should be pleasant, relevant to explain the concepts and make teaching alive experience.
3] Change in interaction style	The classroom interaction pattern must be changed constantly to make the class lively and everybody participate in the learning process. Change in interaction style may be in the following four types: 1] Teacher to whole class 2] Teacher to group 3] Teacher to student 4] Student to student
4] Change in speech pattern 5] Oral - visual switching	Teacher should very his/her speech pattern depending on the relevance o the concept and to break monotony of the class. It also helps in gaining and maintaining students attention and in reflecting the importance of the concept being expressed. Speech pattern may be in the following **ViZ,■■ : : ■ ■ . ■** 1J Pausing 2] Low pitch 3J High pitch
	According to the topic, it is necessary to shift sensory channels of students. Oral-visual switching may be in the following ways: 1] Verbal to visual 2J Verbal to verbal - visual 3] Visual to verbal 4] Visual to verbal - visual
6] Pupil activity	In the activity based learning, the par f the •dents.
7J Focusing ---------- _	It implies drawing the attention of t! els a particular point which the teacher wishes to emphasize, Focusing may be verbal focusing, gestural f , and verbal-gestural focusing.

Components	**Very good**	**Good**	**Average**	**Poor**	Very **poor**
1] Teacher's Movement					
2] Teacher's Gesture					
3] Change in interaction style					
4] Change in speech pattern					
5] Oral-visual switching					
6] Pupil activity					
7] Focusing					

Skill of introduction

Introduction skill is the skill required to begin the teaching-learning process on a good note. The objectives of the skill of introduction are as follows:

- ♦ Get students attention and their readiness for learning
- ♦ Arouse student's motivation
- ♦ Clearly indicate the learning experience to be provided
- ♦ Suggest ways and means of the approaching activity to be done
- ♦ Review provisions experiences and knowledge and makes its link to the present content.

Component of the skill of introduction

1 j Gaining Attention

2] Use of pervious knowledge

3] Use of Appropriate Device

Components	Description
1] Gaining Attention	♦ By using voice, gesture and eye contact ♦ By using audio - visual aids. ♦J* Changing the pattern of teacher - student interaction
2] Use of pervious knowledge is a must, because it helps to establish integration between the pre-existing knowledge of the learner and the new knowledge that the teacher	♦ Previous knowledge refers to the student's level of knowledge achievement before instruction begins. Use of previous
3] Use of Appropriate Device	♦ In order to motivate the learner, the teacher should make use of appropriate devices or techniques while introducing a lesson. Eg: Dramatization, models, audio-visual aids.

Skill of closure

It is an important skill to achieve closure at the end of the lesson

Components of the skill

1] Consolidation of major points

2] Application of the present knowledge in a new situation

3] Linking the part knowledge with the present knowledge

4] Linking the present knowledge with the future learning

3.2. LINK LESSON

In micro-teaching technique, teaching skills are practiced one by one separately. At a time, only one skill can be practiced. While practiced one skill, the use of that particular skill is maximized and other related skills may also occur inking indirect-role. Skills practiced in isolation have no meaning unless they are integrated in teaching.

Hence after attaining mastery in various skills, opportunity should be given to the teacher trainees to teach in real situation integrating the skills mastered already. So separate training

programme is necessary for this purpose. This programme is called link practice.

Line practice is a bridge between micro-teaching and macro-teaching where micro-teaching skills are effectively integrated and transferred.

These is a big contrast between micro-teaching and macro-teaching. In micro-teaching, there is a scaled down process in terms of class room size, skills, scope of the lesson, time etc. Macro-teaching is practiced under stimulated conditions. In macro-teaching in addition to the existence of macro elements, there are also class room management problems. In link lesson the trainee are given chance of teaching real students.

There are many method for link lesson. On the method sis that after practicing three sub skills separately, the trainee may combine all the three sub skills in a lesson of 10 minutes. He then practices another three sub skills separately and links them. He then combines all the six sub skills in a single lesson of 15 minutes. And so on till the entire sub skills are combined in a macro lesson of 40 minutes and teaching a full class.

Link lesson sessions are arranged with about 20 students for about the normal class period i.e. 20 minutes. The trainee prepares a series of eight short lessons on single unit and teaches each lesson for 20 minutes using appropriate skills particular to the content. The number of lessons used in link practice is flexible but selected topic should be adequately covered. The teaching skills namely 'Set Induction' and 'Closure' cannot be practiced in micro-teaching session in isolation. So in link lesson, the trainees include these skills also. At the end of each lesson the trainee should get feed back about the lesson.

3.3 MACRO - TEACHING

The word 'Macro' comes from Greek work 'Makros which means 'Long / Large'. Macro-teaching occurs when a teacher provides instruction to the entire class at one time for an extended period of time, usually longer than 40 minutes. Macro-teaching is after done in lecture format and may be used to introduce a new concept, or to practice a new concept. Macro-teaching allows a teacher to introduce new information to everyone at once. While that's sometime an advantage, it can be a drawback if several students are performing below grade level and aren't yet ready academically to learn the new concepts. At the same time, macro-teaching can give a teacher an idea of what subjects or concepts she/he needs to spend more time on, as well as who in her / her class needs additional help. Planning lessons at the macro level helps a teacher stay on track so she's able to meet her/ his goals and cover the entire curriculum before the school year ends.

LESSON PLANNING AND ITS USES

The purpose is to help the teachers to decide about the type of lesson most appropriate in various circumstances and plan the strategies and materials accordingly. This lesson also highlights the importance of lesson plan and unit plan, its components and its uses. A good programme of instruction will contain some lessons given to the whole class, others involving groups of students and still others to cater to the individual needs and ability of the students. If teachers concentrate on their daily lessons they really would be drawing out the best in the students helping them to develop, progress and achievement.

Lesson Plan

Carter V. Good defines a lesson plan as "a teaching outline of the important points of a lesson arranged in order in which they are to be presented. It may include objectives, points to be asked,

references to materials, assignments etc."

Bossing defines, "A lesson plan is an organized statement of general and specific goals together with the specific means by which these goals are to be attained by the learner under the guidance of the teacher on a given day."

Truly it is commented, "Lesson plan is teacher's mental and emotional visualization of class room activities."

Well planned lessons give confidence to the teacher and that is why more emphasis is laid on the daily lesson plan. This ensures progress and continuity. The teacher should answer a comprehensive list of questions based on three basic questions what? Why? and How?

1. What are the goals and objectives I want the students to attain?
2. What am I going to teach and what experiences am I going to provide for the students? and why is it important?
3. How best could I introduce the topic?
4. How do I present and develop the lesson?
5. What teaching aids will I need?
6. What are the various activities that would be appropriate for effective learning?
7. How will I get the evidence of the student, understanding?
8. How do I provide review?
9. How do I evaluate student's attainments and the effectiveness of my teaching?

At the end of the lesson every teacher should ask himself, Are the objectives realized? If not, why? What points from this lesson I should bear in mind in preparing my next lesson? Could I use some of the features in this lesson for future reference?

Functions of a Lesson Plan

The daily lesson plan specifies the area of classroom activity. It serves as a check on the possible wastage in time and energy of both the teachers and the taught. It gives an opportunity to think and be imaginative. It helps the teachers not to deviate from the right direction unnecessarily.

Principles of Lesson Planning:

1. Formulation of objectives.
2. Deciding and selecting the content to be covered.
3. Collecting and selecting suitable information.
4. Presenting of the subject matter in an organized, systematic and effective manner.
5. Participation on the part of the learner as a co-share in the educative process.
6. Assessing the outcomes of the lesson and the achievement of the objectives.

Daily Lesson Planning

While yearly planning is made for the year or session and unit planning is made for the teaching-learning of a particular unit drawn from the prescribed syllabus of the year, the daily lesson planning as the name suggest is the planning made for the instructional work carried out by the teacher on a day to day basis. However, the term daily lesson planning is not much in use. It has been replaced by the term lesson planning for conveying the same meaning. Let us try to understand the

meaning and purposes of lesson planning more clearly through the following discussion.

What is Lesson Planning?

The duties of a teacher demand from actual classroom, teaching. He has to teach daily one or more subjects to one or more classes in his school. While trying to perform his duties regarding the classroom teaching, a teacher has to pass through the following phases

1. Pre-active phase of the teaching.
2. Inter-active phase of the teaching.
3. Post-active phase of the teaching.

Where what does a teacher do in the classroom with regard to his actual classroom teaching along with his students are covered in the interactive phase, the theoretical activities performed at the cognitive level by the teacher before the actual classroom teaching are known to be related with the pre-active phase of teaching.

In simple words lesson planning means the planning of a daily lesson related with a particular unit of a subject to be covered by the teacher in a specific school period for the realization of some stipulated instructional objectives. It is a sort of theoretical chalking out of the details of the journey which a teacher is going to perform practically in the classroom along with his students.

Now the work of chalking out the details of such journey or preparation on the part of a teacher for executing the task of actual classroom teaching may be done either at the cognitive level or preferably in the written form by writing a lesson plan.

In this planning a teacher of mathematics may thus have to pay considerations to the following essential aspects

- Broader, goals or objectives of the subject mathematics.
- Setting and defining of the classroom objectives related with the present unit or topic of the subject mathematics.
- Organisation of the relevant subject matter to be covered in the given lesson for the realization of the set objectives.
- The decision about the method of presentation of the subject matter, teaching strategies, classroom interaction and management.
- Appropriate provision for evaluation and feedback.

How to plan Lesson for Teaching Mathematics?

A teacher has to plan for the teaching-learning of a small subunit or portion of the prescribed syllabus of his subject going to be covered in the fixed duration of the classroom period. It is a quite routine affair for him to be done on the all working days in a scheduled period of the school time table.

The educationists and researchers in the field of pedagogy have suggested from time to time appropriate guidelines for the planning of these daily lessons. However, the schedule suggested by the renowned educationist J.F. Herbart in the shape of his famous six steps has remained quite popular for the lesson planning in almost all the subjects of the school curriculum.

The six steps suggested by him for the lesson planning are as below:

Components of a lesson plan

i. Preparation:

The teacher should first of all prepare the students to get new knowledge. He should excite the students in such a way that they feel the need of learning new things by means of stories or through charts, models and pictures or from student's pervious experiences.

ii. Presentation:

The subject matter should be presented in simple and familiar way. It should be done with the active participation of students. Students should not be passive listeners. Questions asked should be relevant, according to the mental level of students and evenly distributed. Aids should be used and black board summary should be developed side by side.

iii. Comparison and Association:

Whatever students learn, they compare it with similar set of examples. **iv. Generalization:**

It should be done with the participation of students.

v. **Application:**

Whatever the students learn should be applied to new situation and unfamiliar facts so that there is transfer of training and knowledge gained becomes permanent.

vi. **Recapitulation:**

This is the last but not in any way the least important step. There the teacher knows what the students have learnt, where they stand and whether the teacher himself is successful in his aims or not.

The educationists like H.C. Morrison and B.S. Bloom have tried to forward their own schemes and guidelines for the purpose of planning the daily lessons, in the name of unit approach and evaluation approach respectively. In our country also some attempts have been made to suggest suitable scheme and guidelines for the planning of the lesson.

Influenced by the above

1. Identifying Data:

It is the beginning step of the lesson planning. Here the following things are written by the pupil teacher in his notebook and on the black board while initiating to deliver his lesson in the class.

I. Name of the pupil teacher or his roll number.

II. Class and section to be taught.

III. Name of the subject [Mathematics is to be written here]

IV. Topic [on which lesson has been prepared]

It should be written just after making announcement of the aim or purpose of the lesson e.g. at the introductory stage when the teacher has made announcement that "today we will study about simple interest." then he may write name of the topic as "simple Interest".

V. Date on which lesson in being taught.

VI. Duration of the period for which lesson has been prepared. Generally it is 35 or 40 minutes.

2. Aid Material:

Here the type of aid material needed for the classroom teaching-learning of the lesson in hand is mentioned. Note that it is only written in the lesson planning notebook No mention of such aid material either verbally or by writing of the board is ever made by the pupil teacher in the class. Usually the following types of aid material may be mentioned by the pupil teacher in his notebook.

1. Backboard, chalk, duster, pointer etc. [the type of aid material essentially needed for every type of mathematics lesson].
2. Concrete material or real objects.
3. Graphical material such as charts, pictures and graph or models.
4. Slides, films, transparencies or computer presentation etc.

3. Instructional objectives:

These are also written in the note book and never mentioned to the students either orally or by writing on the black board in the class. The framing and writing of these class room instructional objectives is a serious task which needs essential knowledge about the taxonomies of instructional objectives related to all the domains of human behaviour and then the methodologies of writing these objectives in behavioural terms. Pupil teachers are advised to go through the relevant chapter of this text for acquiring essential knowledge and skill for framing and writing instructional objectives of the different types of lessons taught by them.

4. Previous Knowledge:

The pupil teacher here mentions in his note book about the type of knowledge, skill and experiences etc. needed on the part of the students of his class for the teaching and learning of the present lesson. These things are also written in the notebook and never mentioned to the students.

5. Testing of previous knowledge and Introduction:

Under this step attempts are made to test the previous knowledge of the students [assumed under the previous step] through some well framed questions or asking the pupils to do certain types of activities, or making them to solve certain problems or putting them in some specific problematic situation so that they may feel an urgent necessity of studying the present lesson [by being enabled to find its solution through the previously acquired knowledge and skills]. In this way while trying to explore the previous knowledge of the pupils related to the lesson in hand, the teacher tries to make them arrive at the necessity of studying the very

lesson. All the questions and activities related to this step are also meant for the writing on the notebook, and also their help is actively sought for the task of testing the previous knowledge of the students and, making them eager to study the present lesson.

6. **Announcement** of **the** Aim:

Under this step, the necessary announcement regarding the topic to be taught is made by the teacher to the students of the class e.g. today students, we will know about the sum of three angles of a triangle or let us know how the sum of three angles of a triangle is equal to two right angles etc. After making such announcement, the pupil teacher should write the name of the topic to be taught on the blackboard.

7. Presentation:

This step is related to the planning of the real teaching-learning task performed in the class. What type of teaching-learning activities will be performed by the teacher and pupils in the class for the realization of the stipulated behavioural objectives are properly mentioned under this step. In the beginning, the contents to be covered or the learning experiences to be given are mentioned as briefly as possible. After them the activities performed by the teacher and pupils are mentioned under two separate columns named as teacher's activities and pupil's activities. This step really represents the heart of the lesson planning. Here all types of decisions regarding the type of learning experiences provided to the students and the methods, strategies, techniques and tactics used for providing these experiences are properly taken. The questions asked, the responses managed, the mutual interactions among the students and teacher held, the teaching-learning experiences shared, the aid material and other resources to be utilized all are properly planned for the better realization of the stipulated instructional objectives. The planning about all such things find proper mentioning either in term of teacher activities or pupils activities.

8. Generalization:

Many of the mathematics lessons are concerned with the establishment of certain rules, principles formulae etc. Usually the problem solving task related to almost all the topics and branches in mathematics needs proper generalized way of proper solving. As a result in this step, attempts are made by the teacher with the help of questioning device and the illustrations and examples given to help the students in arriving at some or the other appropriate conclusions and generalization.

9. Practice or Drill work:

It is a quite important step for the planning of mathematics lessons and is related with the post-teaching stage. In the lesson planning of other subjects it is known by the term

recapitulation. What is being taught by the teacher and learnedly the pupils is well practiced through the activities performed in this step. The mathematical skills like computational, measurement, surveying, drawing, graphical presentation, problem solving etc. can be properly developed through the practice or drill work, this step is also being utilized for the evaluation of the students learning and acquision of the things taught to them.

10. Homework:

done by them at their homes. It is also aimed for the fixation of the knowledge, understanding and skills acquired by the students regarding the topic taught to them in the class. Usually it may be assigned in the following forms

i. They may be asked to memorize some facts, principles, proof of the theorems and prepositions, ways of solving the problems, formulae and their application etc.

ii. They may be asked to solve some problems either, given in their textbooks or presented to him by the teacher in the class orally or through charts and transparencies.

iii. They may be asked to perform certain practical activities for-making use of the knowledge and skills acquired by them through the classroom instruction e.g. measurement of the total floor,

wall and roof areas of the rooms and other constructions of their own homes and computation of the

iv. Expenses incurred for the white-washing, flooring and paints etc.

In this way an appropriate lesson plan can be properly prepared by a mathematics teacher by following the above mentioned steps for the planning of the lessons in mathematics. However, it should be always kept in mind that a prepared lesson plan howsoever good it may, be, can merely provide some guidelines for showing the path to be followed in the classroom instructional activities. Its implementation rests with the teacher. He has to take right decisions at the right time for making use of these guidelines. These are meant for good teaching and better learning outcomes. These are not meant for following blindly. A teacher should take them as a means for attaining the desired teaching-learning objectives and not as an end in themselves. Therefore, it is always appropriate and advisable to make some or the other changes in the prepared lesson plan while making its use in the classroom instructional work depending upon the situations and circumstances prevailed at the time of teaching and learning.

How should the task of lesson planning be performed by the pupil teachers for the teaching of mathematics may further be well understood through some of the following illustrative lesson plans.

Planning is a must for the successful execution of a task or a project. It not only caters to the proper realization of the aim or purposes of doing that tack but also helps in proper utilization of the time and energy on the part of human and material resources. The same is equally true for the process of teaching-learning. The teachers who plan their work properly prove quite effective in their teaching task. It is why, a mathematics teacher should also concentrate on a wise planning of his teaching and instructional work carried along with his students during the whole session He may have three types of schemes for such planning named as below:

- Yearly Planning
- Unit Planning
- Daily Lesson Planning

Yearly Planning in Mathematics

In such type of the scheme of planning a teacher of mathematics tries to take a complete view of what he has to do in the whole session regarding the instructional work of a particular mathematics class. In this way by a yearly plan we mean the sessional programme, that has to be chalked, out by the teacher in his subject of teaching in the shape of teaching-learning activities to be carried out with his students. For chalking out such a programme a teacher has to take care of the things like below:

1. The total number of working days available for the teaching-learning of the subject during the year.
2. The total number of periods or time available for the teaching-learning of subject during the year.
3. The nature and scope of the subject in relation to the number of topics included in the syllabus, the contents covered in these topics, the type of learning experiences to be provided to the students and objectives of teaching-learning to be realized etc.
4. The means and material available for the teaching-learning of the prescribed syllabus.

Unit Planning

This yearly plan [what is to be done during the whole session] is then further subdivided into monthly, weekly and daily plans for the purpose of proper implementation. There is still another way for its division and implementation which is known by the term unit planning. Let us try to get acquainted with the term unit planning.

What is Unit Planning?

In it simple meaning unit planning stands for the planning of the instructional work of the session by dividing the prescribed syllabus into some well defined and meaningful units.

For more clarity let us first define the term unit.

The term unit has been variously defined as under Carter V. Good. "Unit may be described as an organisation of various activities, experiences and types of learning around a central problem or purpose, developed cooperatively by a group of pupils under teacher-leadership." [1959].

H.C. Morrison. "A unit consists of a comprehensive series of related and meaningful activities so developed as to achieve pupil purposes, provide significant educational experiences and result in appropriate behavioural changes. [1961].

The analysis of the above two definitions may reveal that

- The contents of a unit are always woven or organised around a central problem or purpose.
- Students may cooperate the teacher in the formulation of units.
- A unit consists of well integrated meaningful wholes capable of providing useful learning experiences to the students for achieving the desired teaching- learning objectives.
- The subject matter or content of a unit represents continuity and comprehensiveness in conveying a proper sense or understanding of a particular problem, theme or knowledge area related to a curricular subject.
- A unit represents a wholesome and complete sub-division of the contents of a syllabus quite useful and meaningful in providing rich educational experiences.

In this way by the term unit we may understand one of the complete and meaningful sub divisions of prescribed course of a subject, centered around a single problem or purpose capable of helping in the realization of the desired teaching- learning of the subject. After grasping the meaning of the term unit now we can attempt for knowing about the nature and meaning of the term unit planning. As pointed out already the syllabus for its proper coverage in a session is divided into some complete and meaningful sub-divisions known as units. Now to think about a proper way for the teaching-learning of the subject matter or learning experiences contained in these units by keeping an eye over the proper realization of the teaching- learning objectives of the subject mathematics is known by the term unit planning in mathematics. In this way the term unit planning may be defined as a scheme or plan chalked out for the teaching-learning of a particular unit, [a meaningful and complete subdivision of the learning experiences to be given in the whole session] mentioning the ways and means of imparting learning experiences related to that unit in view of the proper realization of its teaching-learning objectives.

Units Formation in the Subject Mathematics

The task of unit planning in Mathematics starts with the formation of proper units out of the subject matter and learning experiences to be imparted to the students during the whole session. For the formation of units a teacher may proceed as under.

1. He may accept the various topics as mentioned in the prescribed syllabus as different units for the task of his unit planning.
2. He may combine the different alike topics of the syllabus for the formulation of units i.e. Area of all types of geometrical figures, volume of all types of bodies and figures or even more broader like mensuration or Graphs.
3. He may formulate the units on the basis of similarities or symmetry observed in the structure, principles and system of mathematics like Number System, Decimal System, Rational and Irrational numbers, system of integers, Measurement systems, Congruency, etc.
4. He may organise the subject matter into units on the basis of mathematical formulae and principles or similarities found in the solutions of mathematical problems like Algebraic formulae, Simple equations, Simultaneous equations, Quadratic equations, Trigonometric formulae and functions etc.

For the purpose of dividing the contents of the syllabus, into various units, a teacher of mathematics should try to take note of the following things.

i. The total days and working hours [classroom periods and other extra time] available for the teaching of Mathematics in a particular grade or class.

ii. The completeness and meaningfulness of the units formulated in terms of some special purpose or objectives achieved.

iii. Suitability in terms of the age interest, needs and abilities of the learners.

iv. Suitability in terms of the resources and teaching-learning conditions available for the teaching-learning of the units.

v. The proper division of the whole syllabus [contents and learning experiences] in view of the total time and resources available.

vi. Being in perfect tune with the realization of the teaching-learning objectives of mathematics.

vii. The proper integration and correlation of the subject matter and learning experiences available within the units themselves.

viii. The proper correlation, coordination and integration among the different units formulated out of the prescribed syllabus for the needed continuity and convenience.

How To Proceed Further In Unit Planning?

After organising the subject matter and learning experiences into some complete and meaningful subdivisions i.e. units, the further work of planning may be undertaken in the following ways.

1. A unit in hand should be further divided into some suitable sub-units or parts. A sub-unit or part, as far as possible, should contain that much of the subject matter or learning experiences as could be covered within the available classroom period of 35 or 40 minutes.

2. Objectives related to the teaching-learning of the unit should be predetermined and properly framed by expressing them into behavioural terms for making clarity about the types of behavioural changes expected from the students after going through the unit.
3. Proper decisions should be taken about the methods and technique used, and audiovisual aids and other material utilized and teaching-learning experiences given for the realization of the set teaching-learning objectives.
4. Decisions should also be taken about the type of interactions among the teacher and students and the relative roles played by them in performing the various activities during the teaching-learning process.
5. Proper decision should also be taken for the evaluation of the teaching- learning of the unit covered. For this purpose it is always better to prepare before hand the desired unit test. The time and resources needed for the administration of unit test should also be well decided.

Let us now illustrate, the procedure of unit planning in mathematics by an example. An Example of Unit Planning

1. **Subject:** Mathematics
2. **Name of the Unit**: Various types of measures and their measurement
3. **Name of the sub-units:**
    a. Length and its measurement
    b. Mutual conversion of the bigger and smaller units of length.
    c. Weight and its measurement
    d. Mutual conversion of the bigger and smaller units of weight
    e. Temperature and its measurement
    f. Mutual conversion of the measurement of temperature from one scale to another
    g. Time and its measurement
    h. Mutual conversion of the bigger and smaller units of time
    i. Our currency and its units of measurement.
    j. Mutual conversion of the bigger and smaller units of currency.

4. Teaching-Learning Objectives of the unit:

a. The following types of behavioural changes are expected from the students as a result of teaching-learning of the above unit.
b. Students tell what they mean by measurements related to length, weight, temperature, time and currency.
c. Students tell about the utilities of these measures and their measurement in day to day life.
d. Students tell about the units bigger as well as smaller, employed for the measurement of these measures.
e. Students, demonstrate the skill of mutual conversion of the bigger and smaller units of the measurement of these measures.

5. Decision about the methods and aid material:

For the attainment of objectives through the teaching-learning of the contents related to various

sub-units, the required pre-planning for the selection of appropriate methods and aid material etc.

6. Evaluation Work:

At the end of the teaching learning work related to various sub units resulting outcomes will be evaluated through a unit test. This test will help in, determining the extent to which stipulated teaching learning objectives have been realized. The follow up work in the form of remedial education or necessary practice and drill work will be done iii the subsequent period.

On the similar lines a teacher of mathematics may plan for the teaching learning of other units in the subject mathematics related to syllabai of more higher classes. For doing so let us assume 'for one's setting for the planning of the unit named as "mensuration or Areas and volumes of different figures." For the planning of this unit he may then advance by subdividing the unit into the following sub-units each to be covered in a single classroom period.

Importance and Advantages of Unit Planning

1. The syllabus in terms of content and learning experiences to be covered in the whole session is suitably divided into units in view of the time available for the teaching of mathematics.
2. Units represent the unified and integrated wholes of the meaningful and purposeful content material and learning experiences.
3. Unit planning lays proper stress on the formulation of teaching learning objectives of the unit in the behavioral terms.
4. In unit planning, a teacher is well informed about the type of methods and strategies used, aid material and resources utilized for the teaching learning of the various sub units.
5. Unit planning helps in the proper organization and sy stematization of the teaching learning process. A teacher has a prior knowledge of the task and'activities to be executed for the teaching and learning of the unit and sub units and such knowledge makes him mentally and professionally prepared for the fulfillment of his obligations as a teacher.
6. The planning of the activities and resources beforehand and division of the content material into complete and meaningful units makes the task of teaching and learning quite interesting and absorbing leaving no scope for the problem of indiscipline in the class.
7. The administration of the pre-prepared unit, test in view of the stipulated teaching-learning objectives helps in the proper evaluation of the teaching- learning task.
8. Unit planning has a proper provi sion for the diagnosis of the learning difficulties of the students and subsequent remedial instruction.
9. Unit planning has a proper provision for the review, recapituation, practice and drill work related to the contents and learning experiences relative to the sub-units.
10. Unit planning paves the way for a proper and appropriate daily lesson planning. A teacher feels quite at home in the task of his daily lesson planning in view of the blue print provided by unit planning.

Demerits and limitations of unit planning.

Unit planning is discredited on account of its following shortcomings and limitations.

1. The division of the contents of the syllabus into meaningful and complete units and sub-

units is not an easy task. The improper and faulty formulation of units and sub-units may

create hurdles in the path of teachers and students for the proper teaching and learning of the subject.

2. Mathematics is a well organised, systematic and sequenced subject. The certain types of information, pre-knowledge and skills are required for the learning of one or the other topics. However, while organising the syllabus into some or the other units, we have to set aside the logical and sequential development of the subject. It creates too much difficulty for the teacher as well as students of mathematics in their respective tasks of teaching and learning.
3. The unit planning puts restrictions on the freedom of teachers. The predetermined objectives, learning experiences, methods and resources, method of evaluations etc. leave little scope for the originality and creativity of the teacher needed in the changed situations and circumstances faced in the classroom environment.
4. The teaching-learning process becomes too much time bound through the adoption of unit planning. Mathematics is a skilled and applied subject that needs a lot of exercises and activities for the practice and drill work. Limitations and restriction of time provided by the unit planning thus may prove a big obstacle for the teaching and learning of this subject.
5. Unit planning may make the teacher too much conscious for the implementation of planned scheme as the planning becomes an end instead of remaining means for the realization of the teaching-learning objectives.
6. The students and teachers have no ways other than to follow the guidelines provided by the unit planning. It makes the teaching and learning as planning centered rather than students centered. Their needs, interests, abilities and capacities are thus may be unnecessarily sacrificed for the sake of following the path laid down by unit planning.
7. The task of unit planning needs expertization as well as labour and time spent on the part of a teacher. Overloaded with teaching and other functionary duties teacher takes too little interest in the proper planning of the units. Absence of needed skill for unit planning, as well as lack of interest and enthusiasm for doing such work, thus may prove a big handicap in the successful realization of the purposes of unit planning.

Shortcoming and limitations listed above may leave an impression that unit planning is not at all beneficial for the subject like mathematics. However, this conclusion is not justified. It is only one side of the picture. A close analysis may reveal that the defects and problems do not lie in the purposes, processes and outcomes of the unit planning. Planning for any task is always aimed for the proper execution of that task. Unit planning in this way

is always aimed first for the proper division of the syllabus into suitable units and then thinking proper ways and means for the reahzation of the stipulated teaching-learning objectives through the meaningful instructions related to these units. The limitations and shortcomings if any thus lie in the system of implementation and not in the planning. Therefore, there should not be any hitch or doubt in the adoption of unit planning for the teaching-learning of mathematics.

Distinction between Unit Planning and Daily Lesson Planning

Unit planning and Daily lesson planning can be distinguished from each other on some of the following grounds.

1. Unit planning is meant for the division, organisation and planning of the prescribed syllabus being covered in the whole session while daily lesson planning helps in the organisation of teaching- learning in terms of a lesson delivered during a classroom period.
2. The scope of unit planning is much wider than the scope of daily lesson planning.
3. The unit planning may be done for the teaching-learning carried out in a classroom period or its duration can be extended to several days but in the case of lesson planning its duration is strictly limited to a single day task i.e. fixed duration of a classroom period of 35 to 40 minutes.
4. The unit planning carries the objectives of the teaching-learning to be done for the whole unit comprising of the various sub-units. These objectives may have quite wider coverage in comparision to the objectives of daily lesson planning strictly limited to the expected behavioural changes brought out in the limited time of the classroom period.
5. A unit planning may give birth to a number of daily lesson planning depending upon the number of sub-units carved out of a unit in hand. Sometimes a subunit may need its handling through two or more days lesson planning depending upon the nature of instructional work needed to be carried out.

HERBATIAN STEPS

John Fedrik Herbat advocated the following lesson plan steps.

1] Preparation
2] Presentation
3] Association or comparison
4] Generalisation
5] Application
6] Recapitulation

1] Preparation

It pertains to preparing and motivating the children to learn the new topic. The mind of the child should be prepared to receive new knowledge. The preparation should not only set the atmosphere for learning, but it should also arrest attention of the students and focus it on the new topic. The preparation may involve.

- Testing the previous knowledge relevant for learning the new topic.
- Integrating the previous knowledge with the new lesson to be learned.
- Capturing the attention and maintaining interest using audio-visual aids, story telling, etc., Arousing the curiosity of the students by creating a problematic situation or posing an intriguing question.
- Announcing the aim of the lesson.

2] Presentation

Actual teaching takes place at presentation stage. Students acquire new knowledge and ideas. For an effective learning out come, the teacher should ensure active student participation by providing a number of learning activities. Teacher can make use of audiovisual aids and Teaching - learning materials to make the learning interesting, effective and meaningful. The teacher should stimulate the mental facilities of the students by asking questions.

3] Association or comparison

The new knowledge becomes more meaningful when it is compared, contrasted, associated and integrated with already existing knowledge. This step is particularly significant in subjects like mathematics where the students have to learn definition, establish principles or generalizations or formulae deducted from already learnt concept, postulates, theorems and axioms.

4] Generalisation

Students are required to establish formula, law, rule and etc., in mathematics. This is possible presenting particular examples and requiring the students to observe and compare for co

5] Application

In mathematics the students have to apply the rules, formulae or generalizations that they have learnt in order to solve problems. By application the validity of the generahzation is tested and verified.

6] Recapitulation

Recapitulation can be done through the following,

- asking questions on the contents of the lesson.
- giving a short objective type test.
- asking the students to arrange the steps in solving problem in its logical sequence.

MODEL LESSON PLAN

Name of the teacher	:	Subject: Mathematics	Date:
School	:	Topic : Menstruation	Time:
Standard	: *IX*	Sub Topic: Sector of a circle	Period:
Strength			

General Instructional objective: The Pupil

1. acquires knowledge about the method of finding the area of the section of a circle.
2. develops understanding of the method of finding the area of the section of a circle.
3. applies the knowledge to solve some problems using the formula.
4. develops skill in a) drawing free hadn sketch of the section of a circle b) computation
5. develops interest by dicussing the given problem.
6. develops scientific attitude towards the sector of a circle and appreciation by making the sector of circles with different measures.

Specific Instructional objectives: The Pupil

1. recalls central angle and formulas for length of arc of the sector of a circle.
2. recognizes the method of finding the area of the sector of a circle.
3. identifies the shape of circle.
4. translates the verbal form of central angle to symbolic form.
5. suggests alternate formula for finding the area of the sector of a circle.
6. analyses the given problem
7. establishes the required relationship
8. generalizes the area of the sector of a circle with length of arc '/' and radius V.
9. does written calculations systematically.
10. discussess the means to solve the problem.

mmon elements or pattern. Thus, students are led to frame a general law or principle.

11. considers the given problem in all its aspects.
12. makes models with the help of paper cuttings.

Teaching Aids:

Colour Papers for cutting sector of circles with different measures.

Specifications	**Content** analysis	**Learning Experiences**	Evaluation
The pupil identifies	It is circle in shape Ring, ball, bangle etc. Area of circle = ror	What is the shape of this picture? Can you give some examples of things that are circle in shape? What is the formula to find the area of a circle whose radius is 'r?	
recalls	Part of a circle within sector.	In the circle what is sector?	
recognizes	To find the area of a sector of a circle with length 1 units and radius r units	Let us find the area of a sector of a circle whose length of arc T units and radius 'r' units	
analyses	Length of the arc i. radius of the circle r, and central angle.	What are the measures needed to find the area of the sector of a circle?	What are the different types of angles
Makes models translates draws neatlv	ZACB is the central angle \	In the paper cutting of circle with 'c' as centre draw to arcs at two different points and denotes it as 'A' and 'B\ Now how can you write the central angle in the form of symbol? How can you represent the above circle in a diagram?	: :■ ;
recognizes	ACB is the sector of a circle D -. A = ----- 7tr D is the c ;ngle	What can you say about the sector of a circle? What is the area of a sector of a circle? Whose centra! angle is 'D' and radius 'r' Units What is'D'?	What is the value of Jt? i ▮ ..... *
Specifications	**Content analysis**	**Learning Experiences**	**Evaluation**
suggests	D 2 r A = ---- xitr —	How can you rewrite the formula in some other form?	

recalls	D , 2 360	What is the formula to find length of arc of the sector of a circle?	
recognizes	A lr A = — 2	Now what is the formula for finding the area of sector in terms of 1?	
generalizes	Yes	Whether the formula for finding the area of sector in terms of T and 'r' is true for all measures of circle?	If radius is 5cm, and the length of the arc is 157t What 'D' is?
		Let us solve a problem. Find the length of arc and radius if the area of sector a circle is 24071 and whose central angle is 150	
-analyses	Given. A=204jt D=150	What is given in the problem?	What is the formula to find 1?
	Length of arc and radius.	What is to be determined?	
	!50 2 2407t = ----X7T 360	Substituting the given details in the area of the sector of a circle, the length of the arc and radium can be found	
does not written calculations systematically	_2407tx360 150JI = V576 = 24cm	Now find the length of the arc?	A D 2 A ------- rtr 360 It is true.
Establishes relationship	D L = -----27tr 360 = — 2rcx24 360 =24		

Review

1. What is central angle?
2. How will you find the arc length and area of the sector of a circle?
3. What is the area of the sector of a circle?

Assignment

1. The length of arc and radium is 87t cm and 10 cm. Find the area of the sector of a circle.
2. Find the length of arc if the central angle of the sector of a circle is 135 and radius 8JI cm

General Instructional objectives [G.,I.O's]

i] Knowledge ii] Understanding iii] Application iv] Skill v] Appreciation vi] Interest vii] Scientific attitude

GIO's Writing:

- The pupil acquired the knowledge of the construction of a triangle.
- Pupil acquires knowledge about the derivation of the formula for area of a circle.

- The pupils acquire knowledge about the derivation of the formula for the perimeter of rectangle.
- Pupil acquires knowledge of the expansion of $(a+b)^2$
- Pupil gains on understanding of the construction of a trapizium.
- Pupil develops an understanding about the derivation of the formula for the area of semi-circle.
- Pupil develops understanding about the derivation of the formula for the area of acquire - angled triangle.
- Pupil gains understanding of the derivation of the expansion of the identity $(a+b)^2$
- Pupil applies their knowledge and understanding in new situations.
- Pupil applies their knowledge and understanding about the perimeter of rectangle in an unfamiliar situation.
- Pupil applies their knowledge and understanding of the mathematical formula in new situation.
- Pupil applies their knowledge and understanding of the expansion of identities with numerals.
- Pupil gains skill in construction of circle.
- pupil develops skill constructing mathematical figures.
- Pupil develops skill of computation by the drawn geometrical figures

Specific Instructional objectives [S.I.O's]

S.I.O's under knowledge:

- Pupil recalls
- Pupil names the different figures
- Pupil recognises the given values

S.I.O's under understanding:

- Pupil identifies the measurements given for construction geometrical figures
- Pupil indicates the different parts *t*
- Pupil evaluates
- Pupil differentiates
- Pupil gives examples
- Pupil substitutes the values
- Pupil explains
- Pupil compares
- > Pupil generalises the different concepts
- Pupil formulates
- Pupil concludes

S.I.O's under Application:

- Pupil analyses the problem
- Pupil select the suitable method
- Pupil judges the sufficiency of the measurements
- Pupil computes
- Pupil analyses the problem
- ♦♦♦ Pupil verifies the result

S.I.O's under skill:

- Pupil takes measurements with speed and accuracy
- Pupil uses the geometrical instruments
- Pupil selects the appropriate geometrical instruments
- Pupil draws the figures accurately and neatly
- Pupil does the calculation

S.I.O's under Appreciates:

- Pupil appreciates that the mathematical formula which helps in daily life.
- Pupil appreciates the symmetry of mathematical regular figures.

S.I.O's under Interest:

- Pupil does the puzzles
- Pupil catches the tricks and shortcuts in mathematics
- Pupil reads and writes articles in mathematics

S.I.O's under scientific attitude:

- Pupil accepts only after logically proved concepts
- Pupil cross checks the results
- Pupils develops the habit of logical thinking
- Pupils accepts their errors.

Teaching aids

Teaching aids are an important means of making mathematics teaching and learning most effective and interesting. Teacher of mathematics needs to explore the full potential of teaching aids for more effective mathematics teaching. The use of teaching aids help in breaking the monotony of the classroom, teaching - learning process as most of the learning takes place through multi-sensory experiences like listening, watching, touching and feeling.

Eg: Cut-outs, Models, Objects, Magnetic board, Charts, Overhead projector, Improvised aids, etc.

Motivation

- Teacher motivates the pupils by testing their previous knowledge by using the teaching aids.
- Motivation can be done through questions, stories, songs, riddle, teaching aids, previous knowledge and etc.,
- Motivation should be done psychologically.
- Motivation is the stimulating factor towards learning of mathematics.

Presentation

- While presentation teacher writes the topic on the black board by inter-relating the teaching aids and the discussion done on motivation.
- Presentation is the important step because, pupils are mentally clean and fresh through motivation so that they can be ready for new concepts.

Application

- Teacher writes or give directions and asks the pupils to do on their own.

- In this step, pupils are getting opportunity to apply their knowledge and understanding in an unfamiliar or new situations.

Recapitulation

- Teacher suits few questions to test the comprehension of the pupils.
- Recapitulation can be done through summarization also by the teacher.

Assignment

- In assignment step, teacher gives simple and innovative follow-up activities related to the day's topic.
- Assignments should be given in order to stimulate pupils creativity and interest.

Unit IV METHODS AND TEACHING AIDS

4.1 Inductive, deductive, analytic, synthetic, heuristic, project, problem solving and laboratory methods of teaching mathematics

4.2 Activity Based Learning (ABL)

4.3 Active Learning Method (ALM)

4.4 Applications of ABL and ALM

- Format of a typical lesson plan based on ALM

4.5 Introduction: Evocation, Recall, Survey

- Understanding: Concept, Teacher and Individual Solving Problems
- Group Work, Presentation
- Evaluation: Reinforcement, Homework, Remedial measures

4.6 Computer assisted instruction

- E-learning, mobile learning

4.7 Importance of teaching aids

- Projected and non-projected aids
- Improvised aids: Paper folding and paper cutting etc.,

4.8 Criteria for selection of appropriate teaching aids

4.9 Use of mass media in teaching mathematics

4.10 Field trip as a teaching technique

4.11 Characteristics of a good mathematics text book.

We are concerned with the methods of teaching. "How to impart its knowledge? How to enable the child to learn it' are the questions to be answered in this discussion. Different methods of teaching have been proposed or propounded by different educational thinkers or schools of thought in education. The knowledge of procedures of the methods will broaden the outlook of the teacher. The choice for him is not to be made narrow. It would be then left for him to decide form his wide information, which of the methods to use and when. Senses are gateways of knowledge. Sensory experiences form the foundation for any intellectual activity. These experiences could be affected by means of number of teaching aids. They provide for a variety of methods. While using teaching aids the teacher should be clear about the importance of teaching aids. This unit helps the teachers to decide about the type of lesson most appropriate in various circumstances and plan the strategies and materials accordingly. This unit also focuses on cooperative learning, individualized instruction which represents one of the effective innovations in teaching learning process. Computer assisted instruction and Dalton plan is discussed. Seminar, group discussion, team teaching and guided discovery are also discussed. After deciding 'why to teach' and 'what to teach', it is proper to think about 'how to teach'. In which way the subject matter and learning experiences, to be imparted, should be given to the pupils so that the set aims and objectives may be properly realised? Keeping

this very thing before him, a subject teacheradopts some special ways or devices for imparting the desired theoretical or practical experiences to his students. These ways or devices are known as methods of teaching.

Success in teaching depends mainly on two factors

i. Mastery over the subject matter, and

ii. Skill in teaching.

The experiences gained about the subject in schools and colleges help in acquiring the former but for the latter a person has to undergo some specialized training so that he may be acquainted with all the modern and techniques of teaching a subject. For the benefit of those who are going to devote themselves for the cause of Mathematics teaching, some of the important methods are discussed below.

4.1. INDUCTIVE METHOD

You may still have in your memory the way of proving binomial expansion like [a +b]n etc. It was named as method of induction—the way of proving any universal truth by showing that if it is true of any particular case, it is true of the next case in the same serial order. Inductive method takes into account the process of induction. While adopting this method students are required not to accept the already discovered formula without knowing how it has been established. They are helped in its discovery by adopting inductive reasoning. In inductive reasoning one proceeds from particular to general, from concentrate facts to abstract rules and. from the special examples to the general formula. The results are always generalized by studying particular concrete cases and examples. If one rule applies to a particular case and is equally applicable to different similar cases, it is accepted as a generalised rule or formula. How this is done may be understood through the following examples.

Inductive Reasoning in our daily life

1. A small child meets the accidental death of his father. Some time later he comes across the death of his playing mate and a year after his neighbor uncle passes away. These are special and particular cases of death but they may lead the child to conclude that "Man is mortal".
2. A child observes the rising of the sun and getting of darkness after the setting of the sun. He observes at a particular day in the beginning when he becomes able to observe the nature's mystery, on the next day, the same thing happens. He collects these particular instances of rising and setting of the sun in his memory and after some time he himself feels that "The sun rises every day and also sets everyday"
3. A child eats a green apple and feels its sour taste. Again on any day he takes another green apple and experiences the same sour taste. These few examples are, enough to make him conclude that "green apples are sour" and hence afterwards whenever he is offered a green apple he at once refuses to eat it.

The method of acquiring knowledge as mentioned above, is known as inductive method. It is nothing but the learning from direct experiences. Here conclusions are based on the repetition of a particular kind of experience at so many times. Scientists and inventors usually adopt this procedure in order to derive some general law by performing a number of experiments. For example 'the law that matter has weight' has been discovered only after weighing so many types of matter at so many places.

Teaching Mathematics with the help of Inductive Method

1. Knowledge of the sum of three angles of a triangle:

Children in a class maybe asked to construct a few triangles of any size or shape. Then they may be asked to measure and sum up the angles in each case. They may find that the sum is the same in all cases. It may lead to conclude that the sum of the three angles of a triangle is equal to two right angles.

1. Establishing the Formula:

$(a + b)^2 = a^2 + b^2 + 2ab$. Students may be asked to find out the square value in each of the cases like $(a.+ b)^2$, $(x + y)^2$, $(m + n)^2$, etc. by the simple method of multiplication. After doing these different multiplications, they may be helped in generalizing that $(1st\ term + 2nd\ term)^2 = (1st\ term)^2 + (2nd\ term)^2 + 2\ (1st\ term) \times (2nd\ term)$.

DEDUCTIVE METHOD

In deductive method one follow deductive reasoning which is just opposite to the Inductive reasoning as may be seen through the following examples.

1. When the child comes across a case of death he may be told on enquiry that on one day or the other one has to depart. He may have verification of this established fact after coming across some other cases of death in his life.
2. The child may be told that he should never eat the green apples because they are sour. Afterwards he may verify this fact by tasting some green apples.
3. In this way deductive reasoning begins with the deduced results or generalised conclusions. Therefore in deductive method, one proceeds from general to particular, from abstract rules to the concrete cases and from the general formula to the special example. In this method the students are told to accept a generalized truth or pre- constructed formula as a well established truth and then asked to apply it in solving so many particular relevant problems.

How deductive method is used in the classroom for teaching mathematics can be understood through the following examples:

1. The teacher may announce that today he is going to teach simple interest. He will the give the relevant formula i.e.,

   $$S.I = PNR/100$$

2. For acquainting the students with its application he may also solve a few problems. Then, he may ask the students to solve similar problems directly with the help of the given formula.
3. The teacher may tell the students that the sum of the three angles of a triangle is equal to two right angles. Afterwards students may be asked to verify this established fact by measuring the angles of the different triangles.
4. Student may be told about the formula of the area of a rectangles i.e. A = Breadth x Length and then be asked to apply it in finding the areas of different rectangles.

Which Methods—Inductive Or Deductive—Should Be Used

The above discussion may lead us to conclude that inductive method is a method of establishing formula or deriving generalised results from the particular examples, direct experiences or experiments. It is a method of discovery whereas deductive method is a method of applying the deduced results. Here, although the students do not get opportunity for original thinking and.

discovering, yet it is very much useful in saving their time and labour. In this way both the methods have their own advantages and disadvantages. Therefore an attempt has been made to combine both these methods in such a way as to derive maximum advantages by doing away with the possible defects of both the methods.. This method in combination—is named as Inducto-deductive method. In adopting this combined method, beginning is made with the inductive, method. The students are made to discover truth or establish a formula with the help of inductive method.

Later on, deductive method is used for the following:

a. to provide opportunity for the application of the generalised formula.
b. to memorize the formula through drill and practice work.
c. to develop skill, speed and accuracy in the process of computation.

In this way Inducto-deductive method requires the combined use of inductive and deductive reasoning.

ANALYTIC METHOD

Analytic and Synthetic methods are two such methods which in spite of their opposing nature are used in combination. Let us try to study these methods one by one.

What is analytic method?

Etymologically the word 'analytic' has been derived from the word 'analysis' which means to take apart or to separate the things that are together. In other words, analysis is a process of breaking a thing into its smaller parts. By analysing a problem, thus, we mean to break the problem into simpler elements or unfold its hidden aspects in such a way that its solution may appear quite obvious. In this method, one moves from unknown to known by adopting the process of analysis. The beginning is always made from the conclusion or what is to be proved and then by operating it analytically the unknown is ultimately linked with the known.

SYNTHETIC METHOD

What is synthetic method?

In contrast to analytic method, synthetic method takes into consideration the process of synthesis. In synthesis, the smaller constituents or parts of a thing are combined or put together so as to give something new. Synthetic method leads us from known to unknown as the known bits of information are synthesized for reaching the unknown. What is already given or known is arranged in such a way that the synthesized structure may lead us to the desired results or conclusion. Here the start is always made from the hypothesis and not with the conclusion as in analytic method.

How can you further simplify it?

acd + 2b2d= bc2 + 2b2d will be true

If acd = bc2 [canceling common from both sides]

or if ad=bc [doj [Canceling off common from both sides] or acd + 2b2d = bc2 + 2b2d [by cross multiplication] Q. 2. How can you further simplify it? acd + 2b2d= bc2 + 2b2d will be true If acd = bc2 [canceling common from both sides]

or if ad=bc [do]

How would you write this relation in some other form?

Exp. Ans. This will be true if a/b=c/d [which is known and true]

Conclusion

In this way a close analysis may reveal that both of these methods have advantages as well as disadvantages. Therefore, it is unwise to advocate one method by completely discarding the other. The need is to synthesize these methods. In the beginning we should try to use analytic method for making the students discover proofs and solutions and then afterwards synthetic method should be used for the presentation of the discovered proofs and solutions As far as the understanding of something is concerned; it should be acquired with the help of analytic method but for fixing and retaining what has been understood, the synthetic method should be brought into use. In this way analysis and synthesis both should be used in combination. Analysis must take lead and necessarily be followed by synthesis for realising the better results in the teaching of Mathematics.

HEURISTIC METHOD

Etymologically, the term "heuristic' has been derived from the Latin word 'heurisco' which." means T have found out.' The method as such was originated by Professor N.E. Armstrong. He was of the view that the child must be made to discover things himself. It is of no use to acquaint the students with facts, rather they should be made to investigate or discover the facts. It is not the knowledge but the way or method of enquiry or investigation that should be aimed in the teaching and learning process.

Keeping in view the above ideas he propagated the use of heuristic method. Elaborating the meaning of this method h; writes, "Heuristic method is the method of teaching which, places the students as far as possible in the attitude of a discoverer".

A close analysis of this definition suggests that heuristic method is not any specific or separate method in itself. A broader definition of this method may be evolved as below.

"Any method, which is opposed to 'be dogmatic method of teaching, which can make the students learn or acquire knowledge independently by exercising their thinking and reasoning powers and which can foster the habit of self-activity and self dependence can be called as heuristic method."

In this way, in true sense heuristic method aims to develop a particular type of attitude, named as 'heuristic attitude' among the students. It seeks to bring a change in the nature of learner. He must not remain as a mere passive listener or receiver of the knowledge. From a passive recipient, he must be changed into an active independent enquirer, investigator and researcher. Instead of telling the things he must get opportunity of discovering them. In this way a!i the good new methods like Inducto-deductive method. Analytic-synthetic method, laboratory method, can take the form and shape of heuristic method, provided if the development of heuristic attitude may be properly aimed in the teaching-learning process.

How to use heuristic method?

While teaching through heuristic method teacher puts .a definite problem before the students. Every student is asked to solve the problem or to discuss and investigate the ways and means to solve the problem theoretically or practically as he desires. The main thing is that the students as far as possible have to remain independent thinkers and discoverers. The teacher has to remain very cautious in providing necessary guidance and help to the students. He has to look for the maintenance of heuristic attitude. The students are to be made active independent enquirers and discoverers of the knowledge rather than mere passive recipients. For this purpose the teacher asks such thought provoking questions that may lead the students to independent thinking, reasoning and

striving for the discovery of the solution of the problem: The teacher in this way only provides a suitable direction to the students to proceed independently for the solution of their problem.

Let us try to illustrate the procedure adopted by solving a. problem on profit and loss.

Problem: Ram Singh bought a cow for Rs. 500/- later on at his own need he sold it to Jailal for Rs. 400/- Try to find out his loss percentage.

Procedure: The students will be asked to study this problem carefully. Then, the teacher will lead them to the solution of the problem through the suitable heuristic questions as illustrated below:

1. What have you to find out in this problem?

A: Loss percentage

2. How will you find out the loss percentage?

A: First of all net loss will be calculated. It will lead us to compute loss percentage.

3. Try to find out the net loss in this problem

A: Students will discover that net loss to Ram Singh is of Rs. 100/-

4. How will you compute loss percentage now?

A: Students, on the basis of their previous knowledge know that profit or loss always incurs on the cost price. Hence they can readily respond that this lOORs. Loss has incurred on Rs.500/- They also know that percentage is always calculated on 100 and hence they can very well proceed to solve the problem.

In this way the students may be persuaded to solve such more questions and then they may be helped to generalize the following formulae related to computation of loss or gain percentage.

Loss percentage = [Net loss x 100]/ Cost Price

Profit percentage = [Net profit x 100]/ Cost Price

MERITS OF HEURISTIC METHOD

1. Psychological method:

Heuristic method is a psychological method in the sense that it is based on the psychological principles. Knowledge is not thrown upon the child, rather he is made to discover the things himself with his own pace. In this method the needs, interests and motives of the child are fully taken into account and hence heuristic method takes the students on the right path of learning through a healthy psychological wave.

2. Development of Scientific attitude:

Heuristic method helps in the development of scientific attitude among children. In this method a great emphasis is laid over the development of reasoning and thinking powers. The students are made independent enquirers and discoverers of the knowledge. It makes them critical and scientific minded. It helps them to be original and creative in their outlook and thinking. In this way heuristic method helps in the inculcation of scientific attitude among children.

3. Emphasis on activity:

Child does not remain passive recipient of the knowledge in this method, rather he becomes quite active for making independent efforts to discover or re-disorder the knowledge

Moreover, this method demands quite alertness and presence of mind on the part of the student for

responding to the heuristic questions asked by the teacher. In this way heuristic method encourages activity on the part of students.

4. Learning through independent efforts:

This method encourages independent learning. The child learns all what he wants to learn through his own efforts. It ensures his mastery over the subject. Instead of spoon feeding the real understanding of the subject matter is acquired. Knowledge thus gained becomes real, lasting and useful.

5. Teacher-pupil Contacts:

Heuristic method gives enough opportunity of developing teacher-pupil intimacy. Teacher has to remain in living touch with the students for giving necessary guidance at the appropriate time. He has to put heuristic questions for loading the students to think and discover the things by their own efforts. For this purpose he has to study every student carefully and know their interests, aptitudes, abilities and limitations. Students, on the other hand, for 'their independent work have to seek guidance and necessary help from the teachers. In this way the success of this method depends upon the nearness of the pupils and the teacher which leads to the establishment of their harmonious relationships.

6. Helpful in discipline:

Heuristic method, as said earlier, needs a lot of mental alertness on the part of students. They are busy in exercising their reasoning and thinking powers and making their independent efforts. They are entrusted with the great responsibility of self-learning and discovering the things and thus have no empty mind for making mischief.

7. No problem of assigning home-task:

In heuristic method the students have to make independent efforts. They remain engaged in the assigned responsibilities even in the class-rooms. This leaves no extra scope for assigning home work and its correction etc. Therefore, no usual procedure of assigning homework is adopted in heuristic method and teacher as well as student get a sort of relief.

DEMERITS OF HEURISTIC METHOD

1. Too much expectation from the students:

This method expects much from the students in demanding that they should discover the things with their own independent efforts. To discover or to explore a new field is not a joke. It needs a lot of hard work, patience, deep concentration, sound reasoning and thinking powers and creative abilities. Every child is not supposed to possess such qualities.

2. Too much expectation from the teachers:

Heuristic method also expects too much from the teachers. They have to frame appropriate heuristic questions for stimulating thinking in the students and provide them individual guidance and help at the proper time. Every child of the class is to be properly studied by them in terms of his basic potentialities and interests. In a class of sixty to seventy students this type of individual attention and guidance becomes almost impossible for the teachers with their limited resources and average abilities.

3. Obstacle in the progress:

Heuristic method requires that the students should discover or rediscover the things. In this way much of the time is wasted in discovering the already discovered facts. This actually creates obstacles in the progress. Human civilization has stood on the legs of imitation and invention. With

the absence of one of the legs imitation, the progress of learning would be surely hampered.

4. Difficulty in covering the syllabus:

Heuristic method is in fact a slow and time consuming method as the students have to go ahead with the self-discovery. It creates difficulties in covering the lengthy syllabus in time.

5. Possibility of erroneous conclusions:

When the students discover the things by themselves, it cannot be safely said that they would always be on right lines. Due to lack of experience and proper abilities, they often draw erroneous conclusions or inferences out of their observations and independent work which in fact proves very harmful to them.

6. Lack of proper aid material:

The successful use of heuristic method requires adequate laboratory and library facilities. There is lack of such facilities in our schools. Moreover textbooks written on heuristic lines are not available. Lack of such facilities creates potential difficulties in the use of heuristic method.

Conclusion

The merits and limitations described above reveal that in spite of being a good method heuristic, method suffers from so many practical difficulties and handicaps. Too much time is wasted in learning a thing and hence joy of discovery, in spite of its greater value, loses its significance. The knowledge gained, although real and useful, remains quite inadequate and insufficient. Moreover the conditions prevailing in our schools do not favour use of this

method and hence it is not possible to use heuristic method in its pure form. The spirit behind the method is quite constructive and useful and hence it should be preserved. The students should not be told everything. They should be encouraged for independent thinking and making conscious efforts to learn by themselves. But on the other hand, children should not be forced to discover everything and left unguided and undirected. The real thing is that we should try to avoid extremes and concentrate on the inculcation of heuristic and scientific attitude among the children.

PROJECT METHOD

Project method is the outcome of the pragmatism-ideas propagated by Sir John Dewey. "What is to be taught should have a direct relationship with the actual happenings in life," this central idea forms the basis of project method. The principle of correlation has been given a very practical shape through this method as it tries to impart education of all the subjects in an integrated way by correlating them with the real life activities. In order to understand this method let us try to think over the meaning of the term 'project'. It has been defined by the different educationists as under:

According to Stevenson "A project is a problematic act carried to completion in its most natural setting."

According to Kilpatrik "A project is a whole-hearted purposeful activity proceeding in a social environment."

Ballard says, "A project is a bit of real life that has been imported into school."

These definitions clearly reveal the following characteristics of a project:-

1. A project is an act related to actual life activities.
2. It is that activity which is undertaken to solve an emerging or felt problem or to realise some useful and purposeful objectives.
3. It is always completed in a social environment and natural setting.

4. It is such act which is most interesting and absorbing.

How to work with Project Method?

Project method gives opportunities to teach various subjects of the school curriculum through working on a suitable well selected project. Essentially the following six steps are involved while working on a project:

1. Providing a situation
2. Choosing and purposing
3. Planning of the project
4. Executing the project
5. Evaluation of the project
6. Recording of the project

1. Providing A Situation:

In this step, a situation is provided to the students to think over in choosing some project to work on. They may be confronted with a problem while studying in the classroom, participating in co-curricular activities and going on excursion etc. It may force them to think about for choosing some project.

2. Choosing and purposing:

In this second step, students try to choose a definite and appropriate project keeping in view of the resources in hand and the nature of the problem faced in the first step. They are properly guided by their teachers in this selection task. Then the aims and, objectives of choosing such a project are properly discussed through group participation.

3. Planning of the project:

The project chosen is again discussed in terms of laying down a plan and procedure for the execution of the project.

4. Executing the project:

In this step, students are engaged in the execution of the project in a natural way without involving any artificiality. They play their roles according to their abilities and capacities with a true social and cooperative spirit.

5. Evaluation of the project:

In this step, the work done on the project is evaluated from time to time. The line of action and mode of execution may be modified on the results of such evaluation.

6. Recording of the project:

This step is concerned with the task of recording. How the project was chosen, planned, and executed, what type of difficulties were faced and how they were solved? How far the project work achieved the desired aims and objectives? All these things; noted down by the students, are properly recorded for the future guidance. The working on a project, while going through the steps given above, requires an extensive knowledge of the various subjects. In a project, therefore, emphasis is laid over in the integration of various areas of knowledge.

A project as defined earlier, is bit of real life and therefore, knowledge of all the areas and aspects is being used in the implementation of a project without caring for the name and nature of a subject. The essence of teaching by the project method lies in the proverb "necessity is the mother of invention". The knowledge of various subjects is given at the time, then and there, when its need is strongly felt while working on a project. Therefore, project method proves to be a way of incidental teaching. Like other subjects mathematics is also taught through such incidental teaching. While working on a project when and where the need of a particular type of mathematical knowledge is felt it is given then and there irrespective of its sequence simplicity or difficulty. This is how mathematics is taught through this project method.

Below we illustrate this type of teaching through an example.

Project :-

Laying out flower garden in the school compound.

One day students go on a picnic to some nearby garden. They happen to see beautiful flowers growing everywhere in the garden. They wish that such flowers should also be there in their school. They convey their feelings to their teacher in charge. Now the problem how to have such flowers in the school is discussed in all aspects. Some students propose to purchase some of the flower pots from the garden. Some say that the services of a gardener should be taken to plant the flower garden in the school. In the end one of the proposals that the students should themselves lay out a flower garden in the school compound is accepted.

Now the prospect of the execution of the project is discussed in a very democratic way. The work to be done is planned well in advance. The students in the group are entrusted with various tasks of the project according to their abilities, interests and capacities. Some engage themselves in studying literature about the growing of flower plants. Some take responsibility of taking necessary guidance from the expert gardeners. Some go to bring seeds and nursery plants, some try to look after the cultivation, manuring of the soil, watering and safety of the plants and others try to keep account of all the expenditure and income involved in the, project. In doing all these activities concerning the project, students get enough opportunities for the incidental teaching of various subjects. In the following lines we would try to think over the opportunities for teaching mathematics through the project in discussion. The activities like the survey of the land to be used for laying out flower garden, division of land into small square, rectangular, triangular or circular pieces, laying out the hedge or wire for safety purpose provide opportunities for the teaching of the topics related to area and mensuration. Number sense can be developed through counting of flowers and plants. Four fundamental rules and multiplication table may be learnt through counting of plants in rows or number of flowers in the plants. Besides this, the accounts concerning all types of purchases, sales, income and expenditure may provide enough opportunities for the learning of these topics. The activities like division of work, maintenance of accounts and various records provide opportunities for the learning of the topics time and work, profit or loss, average, partnership, simple and compound interest etc. In arranging the plants into some organised form, to make way for watering and putting hedges and wires many of the topics concerning geometry may be taught.

In this way the project laying out a flower garden offers so many opportunities for the learning and making practical use of so many principles and facts of mathematics. Besides learning mathematics, it provides: enough opportunities for the learning of the other school subjects like history, geography, Science, language and crafts and thereby present a clear picture of the integration of the knowledge in a very natural, simple and practical way.

MERITS OF PROJECT METHOD

1. Psychological method:

Project method is based on psychological principles. The innate tendencies, interests and aptitudes of the students are best utilised in this method. The instincts of curiosity, creativeness and hoarding also get their satisfaction in project method. In fact it is a play way method of teaching where a useful interesting and purposeful activity becomes the centre of teaching-learning process. This method also suits the modern theories of learning and hence it can be termed as a psychological method.

2. Democratic way of learning:

Liberty, equality and fraternity are some of the cardinal principles which form the basis of project method. Right from the selection of the project till its execution students are provided sufficient freedom for thinking, decision making and going ahead in their assigned tasks. All of these students cooperate in a common project according to their tastes, temperaments, abilities and capacities. In this way this method teaches a good lesson of democratic living.

3. Development of social virtues:

So many virtues essential for good citizenship like self confidence, tolerance, patience, self-dependence, self-respect, sense of responsibility, duty boundness, resourcefulness, mutual love and cooperation etc. are inculcated through project method.

4. Practical method:

This method of the maxims of teaching like 'learning by doing' and 'learning by living'. Therefore, what is to be learnt in project method is learnt by doing that in a very practical way. In project method the problems concerning actual life activities are undertaken. Therefore, it provides sufficient training lo the students to use their learning in their practical life.

5. Dignity of labour:

Project method emphasizes dignity of labour. Students irrespective of their caste, creed and social status join their hands in 'doing mental and manual labour in the execution of the project. A little apathy or slackness on their part towards doing their duties may bring disappointment to them. Therefore, they learn value of the work through this method.

6. Correlated teaching:

Project method presents an ideal picture of the correlated teaching. Teaching of mathematics while having integration with other subjects of the school curriculum can be properly linked with the work experience areas and day to day life activities. The project itself belongs to the very realm of physical and social world and has a direct link with the commonly felt problem of the student Moreover, it is completed in its most natural setting and thereby knowledge is imparted as a unified whole. In this way project method provides good opportunity for, the correlated teaching.

7. The end of various educational problems:

Project method presents incidental way of teaching. Therefore, there is no need of time table construction in this method. The problem of indiscipline is also automatically solved as the students are completely absorbed in their tasks. Moreover, no home-task is assigned in this method because the work in, hand is completed in a co-operative way in this method. This method also provides relief to the authorities as they have not to arrange big classrooms, furniture's and other costly aid material. Here the

actual life situations and setting in the physical and social environment may become real platform for the dissemination of knowledge. In this way so many educational problems are automatically solved in teaching through project method.

DEMERITS AND LIMITATIONS OF PROJECT METHOD

1. Difficulty on the part of teacher:

Project method provides many challenges to the teachers in charge right from the selection of the project till its execution. Every teacher is not equipped with such enthusiasm, abilities and leadership essential for working with such a method. It expects that teacher must be a walking encyclopedia having an all round knowledge of every subject along with its practical application in. day to day life. No teacher is supposed to have a theoretical and practical thorough command over all the subject of the school curriculum and hence this method suffers from serious handicaps in terms of its practical application.

2. Uneconomical method:

Project method is an uneconomical method in the sense that the time, labour and amount spent in this method is quite larger than the return received. Students, irrespective of the guidance given to them, waste so much energy, time and money due to their lack of experience and immaturity.

3. Not suitable for teaching Mathematics:

No organised and systematic teaching is possible in the project method as it provides incidental teaching. But Mathematics needs well organised and systematic teaching. It is a sequence subject where all pieces and units a knowledge require an integrated and systematic approach but in project method such provisions cannot be availed. For example let us take the project - Running a Cooperative shop in the School. Now think how can all of the students in a class learn principles and facts of mathematics through such project. It is difficult to teach 60 or 70 students of a class to make them learn thousands of odd combinations of the multiplication tables. They cannot even learn all the facts of profit and loss through the role playing of shopkeepers and customers.

4. Moreover, it is not the end of mathematics:

There are so many branches, topics and aspects of mathematics that may be hardly covered through such projects. Really speaking if we analyse the fruits of project method we can know that project method is not a method of learning the new facts and principles but it is a way of utilising or learning the- application of already discovered learned facts. But here also conies a difficulty that the drill and practice work which is the backbone of mathematics teaching can also no be done properly through the teaching by this method. Therefore, project method provides serious handicaps in learning mathematics.

5. Difficulty in covering the syllabus:

Project method puts obstacles in terms of the coverage of lengthy school syllabus. A particular project for example laying out a flower garden takes so many months in its completion. Through projects hardly a part of the syllabus can be covered and therefore, it does not suit the present day classroom teaching.

6. Not suitable to the present day school conditions:

In India, our schools can neither afford sufficient money nor provide appropriate personnel for teaching with project method. For using this method, suitable text-books are also not available.

Schools are overcrowded and the educational structure is examination oriented. No provision has been made for the project method in our examination system. Therefore, it seems quite impracticable to make use of the project method in our schools. **Conclusion**

In this way project method, irrespective of having so many good points on its credit side, suffers from so many handicaps and limitations. It is clear that in no case project method may be taken as a reliable method of teaching mathematics. Incidental teaching always needs a support from the organised classroom teaching. The present classroom teaching, therefore, cannot be replaced by project method teaching. On the other hand it is also true that a mathematics teacher should have a practical and natural base for his teaching. Mathematics must be taught in the way it is utilised in our practical life. Here lies the need of working with some useful and productive activity covered by the name of a project and a wise teacher should utilise it for the learning of real and useful mathematics.

PROBLEM SOLVING METHOD

Students while studying mathematics have to make constant efforts for finding the solution of the assigned problems. In their day to day life also they are confronted with so many problems requiring appropriate solution. Problem solving is thus an essential skill which needs to be learned by everyone for his adequate adjustment to his environment. It would be better if this skill could be learnt during the school days. Mathematics is full of the opportunities for learning this skill through a wide treasurer of problems, the solutions of which students are made to find out. In this way, problem solving as a method of teaching-learning becomes an urgent necessity in the subject mathematics. Let us try to know about the mechanism and use of this method in the subject mathematics.

Meaning of the term - "Problem Solving"

Usually the term "problem" is used to describe a situation when one is faced with some unknown and asked to find about its identity or it's confined to a place, the existence of which is unknown to him and he feels necessity of coining out. Similarly a student while studying mathematics may be asked some question or is confronted with a mathematical problem given in his textbook the answer or solution of which is unknown to him. At this juncture, the process in which he is bound to be engaged for finding out the answer or solution of the confronted problem or situation may be named as problem solving. In this way the term problem solving stands for "What one does when he does not know what needs to be done." Such problem solving activity is very much essential in the subject like mathematics. The students can very well develop the desired problem solving ability in them by solving various problems in mathematics.

Defining Problem Solving Method

Problem solving method as a method of teaching represents a method which provides opportunity to the pupil for analysing and solving a problem faced by him on the basis of the previous stock of his knowledge enriched with the present means available to him, quite independently by following some systematic and scientific steps and arriving at some basic conclusions or results to be utilised in future for the solution of the similar problems in the identical situations.

The Process Adopted

In problem solving method a systematic and orderly process is adopted for carrying out the teaching-learning process. The process begins with the felt difficulty or problem. The student is then made to think out all the possible solutions of the confronted problem on the basis of what does he know. Inability of finding out the solution with the help of his previous knowledge and experiences makes

him to engage in serious exploration with the help of self-study, mutual discussion and independent practical work. He tries to test one by one the possible alternatives and solutions of his problem and then by his continuous efforts get success in finding out the best possible solution of his problem. The practicability and validity of this solution may be further verified on the basis of its applicability and reliability in the solution of similar problems in other identical situations.

Stages or Steps in Problem Sohing Method

The following systematic steps or stages arc usually employed in following, the problem solving method.

1. Confrontation with the problem:

In this step or stage students are made to face a problematic situation. It may occur spontaneously or is created by the teacher deliberately, As far as teaching and learning of mathematics is concerned, there is no scarcity of such a situation. Enough problems in the form of exercises and assignments are already there in their prescribed textbooks. They may be given problems for the drill and homework by the teachers after the classroom instructions. They may come across with various mathematical problems while studying in the library or doing self study in their homes. The day to day happenings in their surroundings may create many situations requiring mathematical solution. For example, a child may become

eager to know about the computation of the cost of labour for the white washing or painting work done at his home and it may generate likewise problems related to the surveying and measurement of the areas of different figures.

2. Proper understanding of the problem:

Problem should be well analysed and understood before attempting for its solution. What is its nature, what is there to be found out, how is it different from the problems solved in the past, the questions like these need to be properly answered for knowing about the problem in depth. One can, think about the probable solution of a given problem only when he is fully aware about the nature, magnitude and direction of the problem. Therefore, the students should devote enough time for understanding and analysing the given problem. The following things may prove helpful to them in this task.

[i] They must carefully study and think over the problem with the desired concentration, patience and peace of mind.

fii] The problem should be analysed properly as to know what it is and how one can proceed to solve it step by step.

[iii] The problem maybe divided into some small meaningful parts and where needed it should be summarised through numbers, words or geometrical figures for its proper analysis and understanding.

3. Search for the probable solutions of the problem;

After proper understanding and careful analysis of the given problem serious attempts should be made for finding out the solution of the problem. At the first instance, where possible help should be taken from the previous experiences helpful in solving the problem in hand. In case, problem is new and can't be solved on the basis of previous knowledge, skills and experiences, then alternate attempts should be made. One can have required study in the library, perform practical work in the workshop and

laboratory, do surveying, weighing, measuring or any other useful activity, knock the doors of related sources and agency for gathering relevant and useful information, and take help and guidance from the concerned persons including subject teachers and experts. In this way multi-dimensional efforts should be made in search of some possible solutions of the given problem. Where needed trial and error method can also be adopted for listing out these probable solutions.

4. Solution and testing of appropriate solution:

Out of the possible tentative solutions or hypotheses, the attempts are now made to search out the best. For this purpose all the possible solutions are taken one by one, discussed and weighed in terms of their validity and practicability. The most relevant one is then subjected to proper testing. The selection of the most relevant solution out of the so many solutions is made quite cautiously. In general, the following considerations are kept in mind while making such decision.

[i] The selected solution must be able to present a valid as well as reliable solution of the problem.

[ii] The solution should be in tune with the pre-established facts and principles.

[iii] The situations and negative examples that may cast doubt over the validity of the selected solution should also be kept in mind.

5. Utilization of the accepted solution:

A particular solution after proper testing is accepted as a best possible way of finding out the solution of the problem in hand. Now further attempts are made to apply it in the solution of the similar other problems. In case it helps, it may be accepted as a valid and reliable conclusion or solution for being applied in the solution of the problem of a particular nature in a particular situation. If not, further attempts are made to search again for some more reliable and valid solution helpful in solving the problems in hand. In doing s, it is also kept in mind that the solution must work in the real life situations so that the child may learn to find out the solution of the problem not merely on theoretical grounds but also on the sound practical footings.

Importance and Utility of Problem Solving Method

Problem solving carries the following advantages in the teaching of mathematics.

1. Mathematics teaching primarily aims to make the students able to solve mathematical problems with their own independent efforts. Problem solving method helps much in the attainment of this objective.
2. In the study of mathematics students are constantly required to solve different types of problems, learn different methods for the solution of the problems, acquire new information, knowledge, and skills and make use of the acquired knowledge and skills in the practical and applied situations of the life. All such requirements can be easily met by adopting problem solving method for the teaching and learning of mathematics.
3. This method provides valuable opportunity for the development of mental and cognitive abilities of the students. In fact it has no parallel in the task of developing the power of reasoning, thinking analysing, synthesizing, generalizing and making conclusions among the students.
4. This method helps the students in learning the most systematic and scientific method of problem solving.

5. The development of problem solving ability makes the student quite self reliant and self confident in solving any type of problems related to curricular or non-curricular areas.
6. The use of problem solving method proves a good source of the internal motivation to the students. The solution of the problem found out by the independent efforts makes a student further motivated in the task of learning mathematics.
7. This method is helpful in the development of harmonious relationship between the teacher and taught.
8. It is a psychological sound method on account of its being child centered and problem oriented. The children get completely absorbed as they accept voluntarily the challenge of solving the problem with their own independent efforts. What they learn, is learnt with their own efforts and the satisfaction they get after finding out the solution of their problem makes them more enthusiastic and energetic in further learning and solving of the problems.
9. This method is specially helpful in getting rid of many teaching learning problems like the problems of indiscipline, assigning the home-work, etc.
10. This method proves quite helpful in making the study of mathematics more useful and practicable in the day to day life. The students learn to utilize the acquired facts and procedure for solving their day to day life problems.

Demerits and Limitations of Problem-Solving Methods

Problem solving method is said to suffer from the following demerits and limitations:

1. Problem solving method is best suited for the teaching of solution of mathematical problems. However, there lie so many things in the syllabai of Mathematics apart from the teaching of the solution of problems. Knowledge, understanding and skills related to these topics may not be successfully given with the use of problem-solving method. In this way, this method has a partial applicability in dealing with the mathematics contents.
2. Problem solving method requires independent efforts on the part of students to find out the solution of the problems. It demands from them to be trained in the scientific procedure, scientific thinking and problem solving. Every student is not to be expected to possess such abilities and consequently to carry on all the students of the class for the adoption of this method poses a great problem.

. The task of thinking about the possible tentative solution or hypothesis is a quite challenging one. The students are more often tempted to pick up wrong hypotheses or follow the wrong path of solving problem and are thus found to waste time and energy in the useless and irrelevant attempts.

LABORATORY METHOD

Laboratory method of teaching mathematics is that method in which we try to make the students learn mathematics by doing experiments and laboratory work in the mathematics room or laboratory on the same lines as they learn sciences by performing experiments in the science rooms or laboratories.

This method involves the maxims of teaching like 'learning by doing', 'learning by observation', and 'concrete to abstract' etc. It provides a practical base to our inductive reasoning. The present day teaching is criticized vehemently on the ground that it provides a bundle of theoretical knowledge without any practical ground. Laboratory method is quite competent to check this evil. It can help in

learning mathematics in the way it is used in our day to day life. The theory and practice may proceed side by side in this method and hence it may make the teaching and learning process as interesting, useful and lively as possible.

The laboratory method aims to arouse teachers to a belief, not only theoretical but practical and effective as well that mathematical dishes must be made appetizing and palatable. They are to be accepted with pleasure and digested with ease.

J.W.A.Young.

In this way, laboratory method has too much to contribute in mathematics teaching. For its use it needs a well equipped laboratory and a laboratory conscious, skilled mathematics teacher. Emphasizing the need of laboratory in this method Young has remarked that "a room specially filled with drawing instruments, suitable tables and desks, good blackboards and the apparatus necessary to perform the experiment of the course is really essential for the best success of the laboratory method".

What is to be kept in the Mathematics Laboratory?

What is to be kept in the mathematics laboratory should be known to the teachers of mathematics.

1. To teach counting and four fundamental rules teacher may make use of laboratory method by taking help of concrete materials. How many sticks or stones does a student have? The total number of concrete items possessed by these students may be known through actual counting. Similarly subtraction, multiplication, division etc, can also be taught with the help of concrete objects, charts and models etc.
2. Knowledge of multiplication tables may also be given through concrete objects. Let us take multiplication table of 5. Students may be asked to make several bundles of slicks in a group of five each by typing them with the help of rubber band. Let it be arranged in such a way that the first line has only one bundle, the second two bundles, third three bundles and so on up to 10th line. Let the bundles now be untied and sticks be counted by the pupils. They will find that by combining 5 at 2 times one gets 10, combining 7 at 3 times 15 and so on and so forth. In this way students may be properly acquainted with the multiplication table of 5.
3. In geometry much can be experimented and practiced with the help of laboratory method The drawing of straight lines and angles, dividing them into required parts and construction of triangles, quadrilaterals etc. all demands the use of laboratory work and can only be learnt through experimenting with geometrical instruments.
4. For determining the ratio between the circumference and the diameter of a circle, laboratory method will involve the following steps:
   i. Students may be asked to draw circles with a diameter of 8n cm on their cardboard pieces and ask them to cut out such circular figures.
   ii. Ask them to measure the circumference of these circular figures either by
      a. Measuring the length of the thread tied around the figure or
      b. Measuring the distance traveled by the circular figure on rolling it down on a piece of paper while it makes one complete revolution.
   iii. Ask them to compare the measured circumference with the diameter.
   iv. Ask them to repeat the same above experiment with the circular pieces of varying diameter and

note the deduced results.

v. Now help the students to think inductively and establish the fact that

Area of circle/diameter=22/7=5

5. Laboratory method is easily applicable for the calculation of the area of the figures like rectangle, square, triangle and circle etc. as may be seen from the following examples:

a. Area of **rectangle**

a. Ask the students to take a rectangular piece of a cardboard and measure its length and breadth.

b. Ask them to divide the length ad breadth into parts

c. Ask them to measure the area of any one these square pieces.

d. The above experiment may then be repeated with the help of other rectangular cardboard pieces of different dimensions and the student may be persuaded to conclude inductively that

Area of a rectangle =length x breadth.

ii. Area of a triangle

i. Ask the pupils to draw a figure of a triangle on a piece of thick paper and cut it out with the scissors.

ii. Now ask them to find out its weight with the help of a physical balance.

iii. Ask tern to cut a unit area figure from this triangular piece.

iv. Now ask them to determine the weight of this small square piece with the help of a spring balance.

v. Then, the students may be helped to determine the area of the triangle by comparing the weight of a unit area with the weight of the whole triangular piece.

MERITS OF THE LABORATORY METHOD

1. Psychological method:

Laboratory method has a psychological base. It is a child centered method in which the basic interests and natural instincts of the child are given due weightage. Every child is active by nature and has a spontaneous desire to create or manipulate something or the other. Laboratory method suits such psychological requirements of the children and thus proves an appropriate means of learning mathematics.

2. **Follows maxima of teaching:**

Children learn very easily by learning by doing, proceeding from concrete to abstract and from known to unknown etc.

3. Scientific method:

Laboratory method is so much systematized and organized method that it may be safely labelled as a scientific method. The entire steps essential for a scientific method are followed in this method. The

student, after analyzing the problem solves it through experimentation and after necessary verification arrives at a definite conclusion. In this way laboratory method provides the way for systematic enquiry and investigation. New facts may be

explored through this method or the discovered facts may be further testified in similar other practical situations.

1. Clarity and fixation of knowledge:

Senses are the gate way of knowledge. Here the child gets the opportunity of receiving knowledge through his so many senses-eyes, ears and hands. Moreover, the knowledge is earned through ones own attempt. Genuine interest is also developed through practical work. All these things make learning easy, lasting and enduring.

2. Making mathematics useful and practicable:

Laboratory method helps in getting practical and useful information rather than bookish one. What a child learns he learns through doing and hence he finds himself quite capable of using the acquired knowledge in his life.

3. Development of some good virtues:

Laboratory method helps in imbibing some good virtues among the students. Day to day experimental and practical work teaches them dignity of labour and inculcates love for truth and honesty. It also makes them self-reliant and helps them in building their self-confidence.

4. Suitable for individual's capacity and ability:

This method provides sufficient freedom to work according to ones capacity and abilities. The gifted child need not wait for the dullard for his independent experimentation. On the other hand average or even sub-average children can work at their own pace and finish their work at their convenience without disturbing the progress of the gifted children.

5. **Intimate contacts between teacher and taught:**

Laboratory method provides opportunities to bring teacher and the taught closer and closer. Practical work demands individual attention from the teacher. The teacher has to keep watch on the working of students and render appropriate guidance at the appropriate time. Students need to seek help form the teacher now and then in this laboratory work. This makes essential to both of them to draw nearer and nearer. Consequently, the relationship becomes intimate which helps in smoothening of the teaching learning process.

6. Problem of indiscipline solved:

The students are always busy in doing practical work. Their mind is engaged either in the observation of phenomena or outcome of the results. Also the work done suits their interests, urges and capacities. Therefore, there are very remote chances of the emergence of the indiscipline problem in a class taught by laboratory method. **Development of co-operative feelings and social outlook:**

In this method the students get opportunity of working in a group them in the inculcation of group spirit and we-feeling. They develop a social outlook and learn to work and get long with others.

DEMERITS AND LIMITATIONS

1] Partial usability:

Only those topics which can be experimented upon can be taught through this method. Therefore,

laboratory method has a partial applicability.

2] Not too useful in mental development:

Laboratory work is useful in acquainting the students with the facts or learning of some useful skills. But it does httle help in providing opportunity for the proper mental development of the students.

3] Too expensive:

It is a very expensive method on account of the following reasons; **a.**

Maintenance of laboratory:

This method requires a well equipped and properly maintained mathematics laboratory. To keep a separate mathematics laboratory in a school is not a joke. Therefore, the present circumstances do not allow us to work with the laboratory method as it will involve huge expenditure.

. **Need of more stall:**

In comparison to usual lecture or text book method, laboratory method needs sufficient strength of the staff as there is need of paying individual attention for the practical work done by the students. For the maintenance of laboratory, laboratory staff is also needed. The enrolment of the extra staff would definitely bring extra financial burden. Moreover, the mathematics teachers need to be trained on the lines of laboratory method failing which this method cannot be used satisfactorily.

4] **Too much expectation from the pupils:**

In laboratory method it is expected that students must work independently. They should discover and verify the facts like a scientist or mathematician. But how can we expect from every pupil to work as scientists. It is hoping against hope. Immatures cannot be expected to investigate or discover the things independently.

5] Difficulty on the part of teachers:

Laboratory method needs individual attention to be paid to every pupil. All types of students work with their own pace. It is very difficult for the teacher to take care of all types of students and give them guidance at the appropriate time.

6] Wastage of time:

The process of analysis, experimenting, getting results or verifying - is a very slow process.

7] Non-availability of appropriate text-books:

Text book written on the lines of laboratory method is not available. **8] Non-practicable in the higher classes:**

Not possible to teach all the concepts in the higher classes in a concrete way.

4.2 ACTIVITY BASED LEARNING [ABL]

It is one of new teaching learning method. Activity joyful learning approach is a strategy of teaching learning aims at securing maximal participation of students in the teaching learning process. Almost all the children love fun and show interests towards play. Therefore ABL method follows the principles of learning by playing, learning by doing, learning by enjoying and learning by problem solving. This approach required the involvement of multi-sensory organs of the child in the teaching learning process. Much of the researches have been conducted on different teaching strategies relating to joyful learning activities and the results were found to be positive. This approach is unique to attract out of school

children's to schools. The teachers who are involved in implementing this method have developed activities for each learning unit, which facilitated readiness for learning, instruction, reinforcement and evaluation. ABL transforms classrooms into hubs of activities and meaningful learning.

Activity-based learning or ABL describes a range of pedagogical approaches to teaching. Its core premises include the requirement that learning should be based on doing some hands-on experiments and activities. The idea of activity-based learning is rooted in the common notion that children are active learners rather than passive recipients of information. If child is provided the opportunity to explore by their own and provided an optimum learning environment then the learning becomes joyful and long-lasting.

Characteristics of Activity-based Learning

The key feature of the ABL method is that it uses child-friendly educational aids to foster self-learning and allows a child to study according to his/her aptitude and skill. Under the system, the curriculum is divided into small units, each a group of Self Learning Materials [SLM] comprising attractively designed study cards for English, Tamil, maths, science and Social Science. When a child finishes a group of cards, he completes one "milestone". Activities in each milestone include games, rhymes, drawing, and songs to teach a letter or a word, form a sentence, do maths and science, or understand a concept. The child takes up an Exam Card only after completing all the milestones in a subject. If a child is absent one day, he/she continues from where he/she left unlike in the old system where the children had to learn on their own what they missed out on.

Activity-based Learning

The "Joyful Learning" experiment of the mid-nineties had started as an effort to provide special schools for children who had been freed from being bonded labour. Methods and materials, which were devised to help the children catch up on the lost years of childhood, seemed both appropriate and attractive to all children.

The Montessori system has proved to be a tremendous enrichment to ABL. The materials now available in ABL are colourful, easy to handle, hardy and meticulously developed and enable children to understand place value [units, tens, hundreds] and the basic mathematical processes.

By bringing the blackboard from the teacher's eye level to the child's, and by increasing the blackboard space, two more learning aids have been created: a specific space for each child to write and a large space to read each others' exercises. Every child can proudly own a part of that blackboard.

The learning materials are not only systematically stacked on the shelves, but they are colour-coded, for each class level. Also logos of animal and insect forms are used for different aspects of the curriculum. When the child completes one set, there is a card for Self Evaluation. This can be administered by oneself or with the assistance of another child.

In building in the opportunity of recall of learnt material at each stage, evaluation has become part of the process. For the children, there is no failure and therefore, there is no fear of failure. In the conventional school system, so many children drop out of school because they fail! The need for an examination at the end of the school year is made redundant in this system. So easily, has the asura called "Annual Exam" been vanquished!

If we pause for a second, to think of how children are generally given ranks for their performance in school subjects and how ranking becomes a subtle way of indicating the "value" of a child, we have a sense of liberation from ranking here. No child is "better then" or "worse than" another. The teacher

keeps an eye on the levels attained by every child and

sometimes helps by pairing an advanced learner with a slower one, for specific exercises. This kind of peer teaching works well.

It must be noted that the entire system allows for diversity and differential rates of progress. The Achievement Chart clearly shows the positions of the children in each area. Thus the teacher is enabled to track every learner's progress. Monitoring of progress by the teacher is subtly combined with the child's freedom to select the pace of learning.

The ruthlessness of ranking and peer competition is further reduced by mixing the age groups and classes. In a room of 40 children, there could be ten each from Classes 1, 2, 3, and 4. This vertical grouping has several advantages. It recreates a family model, where the older child automatically becomes a guide and helper for the younger one. It encourages cooperation between children, rather than competition among them. We are told that a multi-grade classroom is a problem which many rural schools confront. ABL is a simple solution to that complicated issue. The system absorbs different age groups and different ability levels within the same age group.

Taking the daily attendance is a ritual in most schools, with the teacher calling out the names and the pupils responding. In the ABL method, this process is made child-friendly. There is an Attendance Card for each child, to be filled up everyday by the child. Children love the sense of trust that this procedure implies. When they assemble in the morning, one student from each class level in the room distributes the Attendance cards and collects the filled up ones. The entire process is orderly. It puts the responsibility for marking attendance on the child and not on the teacher. We have all heard about teachers losing their voices because of their shouting and screaming, to keep the children quiet. In the schools with ABL, there are no apparent discipline problems. The structured learning materials have their own logic, which supports the children's involvement in reading, writing and calculating. Children find that they can learn at any speed, without being taunted by classmates or scolded by the teacher. Also, there is no scrambling for adult attention. Discipline is intrinsic to the material and internalized by the children.

The text book is not the only source of knowledge, just as the teacher is not the sole authority. The text book is integrated into the materials. For instance, one of the steps of the ladder contains an instruction to read a specified page of the text book. Clearly, when a child goes step by step on the ladder, his steady progress gives him the skills to read the connected page in the text book. Of course, if he needs help, he could ask the teacher when he is in the teacher-assisted group, or just go across to where she is sitting. Students appear to have no fear of being reprimanded by the teacher. The conventional distance has been bridged here.

Generally, one of the constant problems of schooling is absenteeism. For example, if a child is sick for a week, he cannot follow the lesson when he gets back. He has the feeling of running a race he can never win. However, ABL has a simple strategy to take care of missed classes. The mastering of a skill is not a collective exercise. The child's work is individual. Therefore, he goes to the points on the ladders, where he left off and starts learning from there.

In rural areas, harvest time is when children are needed on the farm. Their short-term absence from school is no longer a problem. Time away from school can be made up. Fairs and festivals can be enjoyed without their seriously disrupting a child's learning activities.

Repetition of a lesson acts as reinforcement. That is accepted pedagogy. But instead of sing-song chanting of tables or whatever, the child in ABL writes on the blackboard first, his notebook next and

finally in the workbook. Since he writes the same material three times, the pattern [be it spelling or grammar], gets well established. Whatever the lesson [names of animals, masculine and feminine nouns or singular and plural words], the strengthening of the connections by repetition, is certainly achieved.

With so many materials directly accessed by the pupils, one would worry about the displacement of the learning cards. However, the pupils have understood that any disorder in the stacking of material is a problem. Each time the materials are returned to the shelf, they are checked out by the child who used it, sometimes helped by another child. It is like the practice of putting their school bags and their footwear neatly on the verandah, which has become second nature to the children. Order and structure in the materials seem to result in systematic habits in the children.

Gender equality seems to have been achieved rather effortlessly. Girls and boys sit at the low desks or on reed mats together and share their work, without any awkwardness. This is particularly important in a culture, where the girl child needs to struggle for equal rights in home and school.

Inclusion is the word that defines an equitable education system, where all children are together in the same school. Children, with disability are admitted into the school and can be seen wearing calipers and participating in the activities. They are fully part of the class and their peers accept them as friends.

Once the ABL system has been mastered by the teacher and the pupils, the burden on the teacher is reduced. Even though the teacher needs a period of un-learning and re-learning, when moving from the conventional system to the ABL, the end result is very satisfying. She is justifiably proud of her mastering the administration of the new system and of the children's achievements Furthermore, up to Class IV, there is no homework. This reduces the teacher's work considerably and frees the young children to continue learning a variety of things from the family and community and from Nature. Knowledge can be garnered from many sources. In this system, the time table is in units of half days. If Mathematics is on the curriculum, the children will be involved with the materials from the beginning of the school day until lunch time. This is done so that the children can concentrate on one subject without the intrusive bell breaking up their lesson. In the conventional school, this would have been tedious for the child, but in the ABL, there is a lot of movement and activity, exchange of ideas and group work. There is no question of boredom.

In the beginning, many parents were skeptical about classes where children of all ages were mixed. It did not look like a school to them. Could the pupils learn well, when they seemed to be enjoying it so much? The parents were invited to come and sit in the classroom. Gradually, as they watched their child's reading skills and general knowledge develop, most parents were convinced that the system worked. The tangible achievements impressed the parents. Many PTA meetings were also held to explain the new methods to the parents. There was Open House on Saturdays, when parents could find out how the system worked. They were willing to suspend their disbelief then, but it was the child's obvious self confidence and self esteem, as well as demonstrable abilities in the formal skills that won the case. Parents soon became allies of the teaching staff.

The key element in the story of the Silent Revolution is the competence of teachers and supervisors in Tamil Nadu, who were exposed to the new system. Some of them, who were trained at Rishi Valley, became the core group. Others were trained by the core group. It was the translation and adaptation of the learning materials that consumed their energy and invigorated them, at the same time. For nearly six months, when a hundred teachers worked on developing materials, working after school hours, from 4.00 pm to 8.00 pm every evening. Their involvement in the process of material

development was total. [[Not surprisingly, their sense of ownership of the method enabled them to work for long hours and strengthened their allegiance to it.

The ABL system has some senior teachers who are experts in handling teacher education and are treated like wise elders. But there are also young trainers, selected to do the training for their enthusiasm and communication skills. A teacher handling Class I or II could have demonstrated a tremendous grasp of the principles and procedures of the new system and be asked]] to conduct the entire training for a rural school. One of the byproducts is a breaking down of hierarchy and a dynamic interchange of personnel, materials and methods.

The ladder is not a mechanical structure with equidistant steps. For each specific academic achievement, different ways of learning are built in. There are several ways of reinforcing learning, while making it enjoyable: song, game, reading, writing and finally evaluating. For the children the most exciting aspect is that they can learn actively and have a sense of fulfillment. The teacher knows the exact level of the child's achievement and can take remedial measures for a child who has slowed down.

There are a few blank steps in every ladder. These are intended for any new area the teacher may wish to include. There is scope for the teachers to be creative. They keep adding songs that the children sing together and stories for the shadow puppet shows. They use familiar material from their own environment.

Apart from all the Corporation schools in the city of Chennai, there are ten schools in every rural block in Tamil Nadu, functioning on this method. They serve as demonstration schools for the entire block. It is expected that the ABL's obvious success in making children competent is the best tool for advocacy.

There is a commonly held idea that children need to be motivated by the teacher or parent, to study. This is not always the case. Children are naturally highly motivated to know and to learn. Most school procedures dampen the enthusiasm of children and suppress their intrinsic motivation. When school methods and materials are devised to be attractive and easy to use, as in the ABL, the inherent motivation of the children is sustained. Learning to learn comes from wanting to learn.

The learning achievements of the children are resoundingly convincing. Indeed, they provide a strong case for the ABL method to be extended to all State-run schools in Tamil Nadu.

Analysis of ABL

One can examine the ABL method and materials through the following five lenses:

a] Clarity of lessons

b] Classroom environment

c] Children's involvement in process

d] Teacher's role

e] Scope for creativity

Clarity of Lessons

Clarity of the lesson is probably the ABL method's most valued asset. The Learning Ladders provide structure as they are planned in a systematic way. The child knows what

must be done next. Each unit of information or process is broken up in such a way that clarity of the lesson is ensured. The method is particularly effective in the fundamentals of Mathematics, as many

children said that it was their best subject. However, there is a need for a review of the language material.

The criteria for the selection of the vocabulary is not too the same in ABL as in the REC of Rishi Valley. There they had built up the set of words to be taught, by listening to the conversations of children around them. For the Tamil kit, the teachers selected letters that are easy to write, made small words out of them and then gradually increased the number of words on the same criterion. For making the ABL English language kit, a different rule has been used. The alphabet is not taught directly, of course, but five words [which begin with each letter of the alphabet] are introduced at a gradual pace. This method has resulted in the addition of words, which are not directly relevant to their everyday experience.

Classroom Environment

Those who have seen the documentary film on the scheme will vouch for the pleasant relaxed, yet disciplined climate of the classroom. There is order in the stacking of materials and in all the procedures that the children follow. The body language of the children shows their enthusiasm. The closing of the physical and psychological distance between teacher and child reflects a very satisfactory feature of the system.

Children's Involvement in the Learning process

There is absolutely no doubt that the children are truly engaged in the act of learning, though there could be degrees of difference among them. During the several hours of observation, one rarely came across a child who was not pursuing an academic task or a related task. Watching the children move into the classes after Assembly was a heartening sight. There was an eagerness in their step and a sense of purpose in their deportment. One is left in no doubt that a feeling of mastery is the best reinforcement for the development of competence. It seems to work far better than external symbols of recognition like 'stars' and 'medals'.

The Teacher's Role

The teacher has a very important role in this system, though it is not obvious to a casual visitor. She has to learn the entire ABL system and work effectively with it. She has to exercise a quiet authority, without becoming authoritarian. An egalitarian attitude may require some un-learning and re-learning for teachers, but when they see it as part of the new culture of education, they are quick to accept it and practice it. They are also able, in this system, to spend some time on children who are slow.

The research team felt, however, that there should be some time allotted to the teacher's voice. For instance, she could read a story or explain a scientific principle. For such an activity, as for sports, it might be necessary to group the children by age. The advantages of the mixed age group have been demonstrated here, but there is also a value for being with peers of one's own age. Some suggestions on doing this will follow in the last section of the report. While teacher domination is not desirable, teacher participation is advisable for at least 30 - 45 minutes a day.

Scope for Creativity

As we noticed there are some blank slots in all the ladders, for the teacher to fill up. This gives her an opportunity for bringing in new material or for including a locally relevant theme. Clearly, there is here, recognition that knowledge is not a pre-determined set of facts. Changing perspectives, new information, the opinions of students and teachers, views of others in the community - all these can and do constitute knowledge. That there is a provision to introduce a new item for study is to be highly

commended. However, a new item is not necessarily a creative addition to the curriculum. Some special monitoring of the items filled in the blank slots would be recommended. The child's understanding of open-endedness to new perceptions may not get enough emphasis, when the materials are presented as an end in themselves. Other ways of allowing children to be creative must be consciously introduced.

Suggestions for Enrichment of the ABL Method

1] India is a country with tremendous diversity in every aspect. When one has a generalization about any fact in India, an exception to it will crop up immediately. In the school curriculum, the experiencing of the vastness of the cultural spectrum must find some place. The 'empty slots' must seek to bring originality and variety. Towards this end, the training of teachers should be strengthened.

2] Music, as it seems to be taught, is a collective effort by children to sing rhymes and songs at the top of their voices. In this activity, there is no sign that much has changed from an earlier era. This needs to be modified and moderated. Children can learn to sing softly, to sing in tune and to take turns to sing. One does not get the idea that they understand what they are singing. There is a sense of enjoyment, of course and that is good, but a feeling of competent singing will be a value addition.

3] Flexibility is allowed in pace of learning and this is a boon. A certain level of flexibility must be available for the occasional re-grouping of children. The practice of forcing children to compete and ranking them according to their performance is shunned by most enlightened educators. And the ABL is quite child-friendly in this respect. Here it is important to see that

having children of the same age together in an activity does not necessarily entail competition. Also it is possible to introduce a small element of competition without hurting anyone, a strategy which has been tried with success. Children of the same age are divided into two or three groups. The quiz question or alternatively, the athletic task is given to the group. Every child must have one chance, but can get help from others in the group.

4] The shadow puppet stories are good. They are simple enough for all children to know the entire dialogue by heart, as we observe from watching a performance. There is scope for introducing other themes for shadow puppets and also other styles of puppetry and dramatization. Hand puppets, glove puppets, finger puppets and a host of other kinds of play materials will bring joy to the children. Drama enables them to cultivate the imagination and enhances their ability to speak clearly and articulately, to express feelings and to convey messages directly and indirectly. Expanding the scope and variety of theatre-based activity is strongly recommended.

5] Every school should have a Dictionary in Tamil and one in English. Children should be taught "dictionary skills". Knowing the order of the alphabets is certainly the first step. Likewise, an Encyclopaedia in one of the languages would be a tremendous asset for their learning. In the ABL, it is not clear what a child, who has completed the ladders, can do with his time. In other words, there must be access to other kinds and higher levels of knowledge. The information ceiling must be raised to provide room at the top.

6] Story books for reading in class and out of class must be provided in large numbers. This should be treated as a priority.

7] The Rishi Valley rural schools, which provided the template for the ABL schools, had one very important part of education i.e. being sensitive to the environment and conserving water, growing plants and creating a green space around the school. That aspect has been totally neglected in the city schools which we visited. Just outside the school room, there was rubble and dying grass. No attention had been

given by anyone in the system, to keeping it clean or attractive. Since manual labour of any kind is totally absent in the set of school activities, it might be a matter to take up after the first rains.

8] Many of the formal sports, which would be ideal for young children, require space and equipment. And lack of funds may be cited as the reason for their conspicuous absence. But athletics can be introduced at very little cost. A good sand pit and a few metres of rope can take care of High Jump and Long Jump. As for running, one needs only some safe space, preferably adjoining the school.

9] This system is better than any other which one would come upon in India, to handle the problem of understaffed schools. The inadequate number of teachers in our rural schools is a constant problem. On one hand, there are thousands of trained teachers waiting to get employment and on the other, there are a number of Primary schools which are short of two or even three teachers. The ABL can be used with advantage, but its success in the long run, will be determined by the children's access to a teacher in the classroom

10] Many a time, we open the newspaper and read about an accident at an level crossing. This, in a country, where the youth are facing unemployment and even their right to 100 days of work a year is the end result of a long struggle by activists. Why cannot we have a match between those who need a job and the obvious vacancy? The question one would address to the Railway Minister is quite similar to the question that we would pose to the Education Minister.

11] The educational scene in Tamil Nadu has many positive ratings to its credit. ABL must build on the strengths. There are many achievements to be proud of, but one cannot afford to be complacent. There must be an annual review of the materials, the methods and the learning processes to ensure success and to reach even higher levels. This educational initiative could well be a forerunner for a positive change in educational standards across the country. We are now at the threshold of a silent revolution.

The Process Of ABL Approach

- Competencies are split into different parts/units and converted into different activities.
- Each part/unit is called a milestone.
- In each subject, the relevant milestones are clustered and linked as chain and this chain of milestones is called LADDER.
- Each milestones has different steps of learning process and each step of learning process is represented by logo.
- Milestones are arranged in a logical sequence from simple to complex and also activities in each milestone.
- To enable the children to organise in groups, group cards are used.
- Evaluation is inbuilt in the system. Separate cards/activities are used for this purpose.
- Each child is provided with workbook/worksheet for further reinforcement activities.
- Children's progresses are recorded through annual assessment chart.
- Each milestone has different type of activities such as introduction, reinforcement, practice, evaluation, remedial and enrichment activities represented by different logos.

Benefits of ABL Approach

- Children learn on their pace.

- Provision of more time for self directed learning and teacher directed learning is reduced considerably.
- Group learning, mutual learning and self-learning are promoted. ♦♦♦ Teachers teaching time is judiciously distributed among children.
- Teachers address only needy children.
- Children's participation in every step is ensured in the process of learning.
- Evaluation is inbuilt in the system it is done without the child knowing it.
- Rote learning is discouraged and almost no scope for wrote learning.
- Periodical absence of child from school is properly addressed.
- Classroom transaction is based on child's needs and interests.
- Freedom to child in learning as he chooses his activity.
- Multigrade and multilevel in learning is effectively addressed.
- No child can move to the next higher step of learning unless attains the previous one.
- Sense of achievement boosts child's confidence and morale.
- Attractive cards and activity create interest among children.
- Scope for child's development in creative and communicative skills.
- Children will have a feel of security as they sit in rounds in the groups.
- Children are allowed to move in the classroom as they choose their activity.
- Moreover the distance between the teacher and the child is largely reduced and the teacher acts as a facilitator rather than teacher.

Yet another silent revolution is in innovative education.

I hear; I forget,

I see; I remember,

I do; I understand.

The education system improvements depends upon the teachers attitude and their interest towards the system, learners, welfare and parents Involvement. These three are the main factors of the educational system. The ABL schema is one of the hallmarking

schemes in the field of education. It paves the way for learning by doing. It includes play way method, demonstration, cooperative learning. It motivates the students of primary school to get involved in the learning activities by themselves. It is quiet an innovative one form the conventional method.

ABL method is very effective and attractive method. Students like this method very much and also they are involved and learn well through this method. The method is newly-introduced, so to see the true colour of it, some more capsule period is required.

Active Learning Exercises

Bonwell and Eison [1991] suggested learners work in pairs, discuss materials while role-playing, debate, engage in case study, take part in cooperative learning, or produce short written exercises, etc. The argument is when should active learning exercises be used during instruction. While it makes some sense

to use these techniques as a "follow up" exercise or as application of known principles, it may not make sense to use them to introduce material. Proponents argue that these exercises may be used to create a context of material, but this context may be confusing to those with no prior knowledge. The degree of instructor guidance students need while being "active" may vary according to the task and its place in a teaching unit. Examples of "active learning" activities include:

- A class discussion may be held in person or in an online environment. Discussions can be conducted with any class size, although it is typically more effective in smaller group settings. This environment allows for instructor guidance of the learning experience. Discussion requires the learners to think critically on the subject matter and use logic to evaluate their and others' positions.
- A think-pair-share activity is when learners take a minute to ponder the previous lesson, later to discuss it with one or more of their peers, finally to share it with the class as part of a formal discussion. It is during this formal discussion that the instructor should clarify misconceptions. However students need a background in the subject matter to converse in a meaningful way. Therefore a "think-pair-share" exercise is useful in situations where learners can identify and relate what they already know to others. So preparation is key. Prepare learners with sound instruction before expecting them to discuss it on their own.
- A learning cell is an effective way for a pair of students to study and learn together. The learning cell was developed by Marcel Goldschmid of the Swiss Federal Institute of Technology in Lausanne [Goldschmid, 1971]. A learning cell is aprocess of learning where two students alternate asking and answering questions on commonly read materials. To prepare for the assignment, the students will read the assignment and write down questions that they have about the reading. At the next class meeting, the teacher will randomly put the students in pairs. The process begins by designating one student from each group to begin by asking one of their questions to the other. Once the two students discuss the question. The other student will ask a question and they will alternate accordingly. During this time, the teacher is going around the class from group to group giving feedback and answering questions. This system is also referred to as a student dyad.
- A short written exercise that is often used is the "one minute paper." This is a good way to review materials and provide feedback. However a "one minute paper" does not take one minute and for students to concisely summarize it is suggested[who?] that they have at least 10 minutes to work on this exercise.
- A collaborative learning group is a successful way to learn different material for different classes. It is where you assign students in groups of 3-6 people and they are given an assignment or task to work on together. This assignment could be either to answer a question to present to the entire class or a project. Make sure that the students in the group choose a leader and a note-taker to keep them on track with the process. This is a good example of active learning because it causes the students to review the work that is being required at an earlier time to participate. [McKinney, Kathleen. [2010]. Active Learning. Normal, IL. Center for Teaching, Learning & Technology.]
- A student debate is an active way for students to learn because they allow students the chance to take a position and gather information to support their view and explain it to others. These debates not only give the student a chance to participate in a fun activity but it also lets them gain some experience with giving a verbal presentation. [McKinney, Kathleen. [2010]. Active Learning. Normal, IL. Center for Teaching, Learning & Technology.]

- A reaction to a video is also an example of active learning because most students love to watch movies. The video helps the student to understand what they are learning at the time in an alternative presentation mode. Make sure that the video relates to the topic that they are studying at the moment. Try to include a few questions before you start the video so they will pay more attention and notice where to focus at during the video. After the video is complete divide the students either into groups or pairs so that they may discuss what they learned and write a review or reaction to the movie.

  [McKinney, Kathleen. [2010]. Active Learning. Normal, IL. Center for Teaching, Learning & Technology.]

- A class game is also considered an energetic way to learn because it not only helps the students to review the course material before a big exam but it helps them to enjoy learning about a topic. Different games such as jeopardy and crossword puzzles always seem to get the students minds going. [McKinney, Kathleen. [2010]. Active Learning. Normal, IL. Center for Teaching, Learning & Technology.]

While practice is useful to reinforce learning, problem solving is not always suggested. Sweller [1988] found solving problems can even have negative influence on learning, instead he suggests that learners should study worked examples, because this is a more efficient method of schema acquisition. So instructors are cautioned to give learners some basic or initial instruction first, perhaps to be followed up with an activity based upon the above methods.

Active learning method: Learning by teaching

An efficient instructional strategy that mixes guidance with active learning is "Learning by teaching" [Martin 1985, Martin/Oebel 2007]. This strategy allows students to teach the new content to each other. Of course they must be accurately guided by instructors. This methodology was introduced during the early 1980s, especially in Germany, and is now well-established in all levels of the German educational system. "Learning by teaching" is integration of behaviorism and cognitivism and offers a coherent framework for theory and practice

4.3 ACTIVE LEARNING METHODOLOGY

As a parallel initiative, Active Learning Methodology has been introduced in VI, VII and VIII classes in all Government and Aided Schools. The classroom process in ALM, inter aha, includes self-study, group study, mind mapping, presentation and discussion by the children with the teacher playing the role of a facilitator. The exercise of mind-mapping of the concepts through self-study provides a lot of scope for kindling and inculcating creativity among the children. As is the case with ABL in primary classes, the classroom processes at upper primary level also have undergone a complete change with children exhibiting unlimited curiosity, interest and enthusiasm for learning. The classroom space has been transformed into a comfortable zone for the children. All children in upper primary classes [Std 6-8] in Tamilnadu underwent a dramatic refreshing learning process, empowering them to break into knowledge systems effectively. It is probably the most rapid transformation of schooling ever attempted. SS A was intensely searching for a pedagogy that would continue and sustain

the focus on the learner, at upper primary level also. Similar to the focus of Activity Based Learning for primary classes, it could spot a pedagogy practiced at The School, KFI, Adyar, and Chennai. This was termed Active Learning Methodology [ALM]. The main thrust of ALM is to support the sure footed emergence of the Life Long Learner, through active engagement of the student in constructing

knowledge. It emphasizes the importance of the engagement of the learner with the sources of knowledge. Students are not seen as were recipients of information from the teacher.

The scope of ALM

The aim of ALM is empowerment of the learner in such a way that he or she is confident and able to function in many contexts. In the middle school years [classes 6-8], such learning can be blended into the curriculum of any school easily. It includes:

- Learning to affirm oneself and one's learning style.
- ♦!♦ Learning to be healthy and safe - Biology curriculum enrichment.
- Learning to think skillfully, recognize and deal with one' s feelings and be resourceful in a variety of situations -Units on Learning for Life
- Learning to live in social systems - living and working together with other people, good citizenship skills and being able to participate in debate and discussion.
- Learning to live in and interact with a physical environment - finding environmentally viable responses in terms of lifestyle and choices.
- Developing leadership and personality among all children in classes 6 to 8.

Above all, Active Learning Skills will help students negotiate the world of knowledge with competence and enthusiasm, confident of their own abilities and opening widening newer avenues to learning.

Enrichment Activities

- Supply of DVDs depicting ALM model classes
- Training and supply of Audio and Video CDs to promote the basic skills
- Supply of modules to facilitate teachers for interesting and challenging homework, which extend the classroom processes to home.
- ♦♦♦ Conducting workshops to promote hands-on learning experiences and experiments.

Simplified Active Learning Methodology

V Std. - Pilot Study ABL is adopted for I to IV standards and ALM is search of pedagogy for Class V that would sustain the interest of the learner and at the same time prepare them for the active learning methodology at upper primary level. The implementation of Simplified Active Learning Methodology in Class V sustains and enhances skills already acquired by the child in ABL. Every child is actively involved in the process of self study, pair study, observation, logical thinking, questioning, small group discussion, large group discussion and linking life situation to class room learning. The special feature of SALM is that the students are able to evaluate themselves [Self Evaluation] which makes the class not only very lively but also ensures the child, where he/she is in the learning process. Above all this, SALM serves as a preparatory mode for the children to cope up with ALM at the upper primary level and also acts as a bridge between ABL and ALM.

Approach of SALM

SALM has its own unique integrated approach, in which the child becomes very,

- Multi-sensorial

- Creative
- Functional **Learning Steps in SALM**

1. Introduction or Evocation
2. Understanding
    a. Guided reading
    b. Individual reading
    c. Underlining unfamiliar words
    d. Discussion in peer group, small group, large group or with the teacher to find out the meanings.
3. Mind map [guided]
4. Consolidation [focused summary]
5. Reinforcement and Enrichment Activities
    a. Framing questions
    b. Small group discussion
    c. Home work
    d. Project
6. Presentation
7. Writing
8. Evaluation

In Mathematics, the brainstorming activities to kindle the thinking skill are noteworthy to be mentioned here. A single query which results in different answers are discussed in small groups/Thereby, the conceptual knowledge of a particular concept is linked with the life situations through the thinking skill.

Scope of SALM

The skills developed by adopting and practising SALM, which bring the leadership quality among the children, are listed below:

- Learning to learn
- Reading to learn and understand
- Learning to think and question
- Learning to relate with life situations
- Learning to live in society - working together with others and participate in group discussions.

It is no doubt, that SALM is so reviving and empowering that the child is an active learner throughout the learning process

Active Learning Method [ALM]

Active learning methodologies require that the student must find opportunities to meaningful talk and listen, write, read and reflect on the content, ideas, issues ad concerns of an academic

subject[Meyers & Jones, 1993].

Bonwell and Eison[1991] state that some merits of active learning are:

- Students are involved in more than listening.
- Less emphasis is placed on transmitting information and
- Greater emphasis on developing student's skills.
- Students are involved in higher order thinking [analysis, synthesis, and evaluation].
- Students are engaged in activities [e.g., reading, discussing, and writing].
- Greater emphasis is placed on student's exploration of their own attitudes and values.

Active learning shifts the focus from the teacher to the student and from delivery of subject content by teacher to active engagement with the material by the student. Through appropriate inputs form the teacher, students learn and practice how to apprehend knowledge and use them meaningfully.

- The educator strives to create a learning environment in which the student can learn to restructure the new information and their prior knowledge into new knowledge into new knowledge about the content and to practice using it.
- Students are assumed to be an intelligent participant in knowledge creation who can lookup definitions before and after class independently.
- Students can develop skills in constructing and using knowledge with the educator's guidance, alone and also with others in small and large groups.
- The educator may explain concepts, principles and methods.
- Visual aids, demonstrations, etc., integrated into class presentations.

- Students have the opportunity to remember upto50% of the content of each class session.
- Students care deeply about their own education.
- Students learn to monitor and discuss their own learning.
- Students collaborate with other students to discover and construct a framework of knowledge that can be applied to new situations.

Scope of ALM

The aim of ALM is empowerment of the learner in such a way that he or she is confident and able to function in many contexts.

- Learning to affirm oneself and ones learning style -ALM classroom
- Learning to be healthy and safe
- Learning to think skillfully, recognize and deal with ones feelings and be resourceful in a variety of situations.

♦♦♦ Learning to live in social systems-living and working together with other people, good citizenship skills, being able to participate in the debate of our times.

♦♦♦ Learning to live in and interact with a physical environment-living environmentally viable response in terms of lifestyle and choices.

Advantages of The ALM

♦♦♦ Active engagement on the child's part

♦ Provides a template for learning and learning to learn ♦♦♦ Te child is not subjected to endless passivity

♦ Applicable in large classrooms and schools with few teachers

♦ Requires no special aids or special equipment

♦ Children can be resources for each other through paired and group activity

♦ The teacher can devote some time to children who need special help

♦ Allow the child to check her/his work against the teachers and thus save the teacher endless corrections while ensuring accuracy in Childs learning.

♦ Works at child friendly and realistic assessment formats

The beauty of the process is its simplicity. Allows room for all children's voices to be heard through discussions and presentations.

4.5 COMPUTER ASSISTED INSTRUCTION [CAI]

Computer Assisted Instruction [CAI] is an advanced technology in the teaching learning process of mathematics. CAI is an improved version of programmed learning. Nowadays computers are integral and inseparable part of education process. CAI, the computer provides instruction directly to the learner and allow them to inneract with it though the lessons programmed in the system. The computers put questions and expects the students to respond. Computers provide instant feedback to the leaner on the basis of his response. Computer is, therefore act as a teacher to the student in this respect. There are many kind of exercises and Practices provide by computer in teaching various aspects in mathematics.

♦ CAI provides self-learning

♦ Learners proceed according to the own interest and peace of learn i ng.

♦ Computers provide unbiared instant feedback.

E-LEARNING

- ♦ E-learning comprises all forms of electronically supported learning and teaching.
- ♦ The information and communication systems, whether networked or not, serve as specific media to implement the learning process.
- ♦ The term will still most likely be utilized to reference out-of-classroom and in-classroom educational experiences via technology, even as advances continue in regard to devices and curriculum.

M-LEARNING

- ♦ The term **M-Learning** is known as "mobile learning 1'.
- ♦ It related to e-learning and distance education

- It is distinct in its focus on learning across contexts and learning with mobile devices.

M-Learning	E-Learning
The term covers: learning with portable technologies including but not limited to handheld computers, MP3 players, notebooks and mobile phones. M-learning focuses on the mobility of the learner, interacting with portable technologies, and learning that reflects a focus on how society and its institutions can accommodate and support an increasingly mobile population. M-learning is convenient in that it is accessible from virtually anywhere. M-Learning, like other forms of E-learning, is also collaborative; sharing is almost instantaneous among everyone using the same content, which leads to the reception of instant feedback and tips, M-Learning also brings strong portability by replacing books and notes with small RAMs, filled with tailored learning contents. In addition, it is simple to utilize mobile learning for a more effective and entertaining experience.	E-learning is essentially the computer and network-enabled transfer of skills and knowledge. E-learning applications and processes include Web-based learning; computer-based learning, virtual classroom opportunities and digital collaboration. Content is delivered via the Internet, intranet/extranet, audio or video tape, satellite TV, and CD-ROM. It can be self-paced or instructor-led and includes media in the form of text, image, animation, streaming video and audio Abbreviations like CBT (Computer-Based Training), IBT (Internet-Based Training) or WBT (Web-Based Training) have been used as synonyms to e-learning. Today one can still find these terms being used, along with variations of e-learning such as e-learning, e-learnins, and e-learning.

4.7 IMPORTANCE OF TEACHING AIDS - PROJECTED AND NON-PROJECTED AIDS-IMPROVISED AIDS [PAPER FOLDING AND PAPER CUTTING]-ITS SPECIFIC USES IN TEACHING MATEMATICS

A teacher has an inherent desire that his teaching should be as effective as possible. What he teaches should be clearly understood, grasped and fixed in the minds of his students. In order to realise his objective, the teacher makes use of different types of aid materials just as charts, models, concrete objects, apparatuses, instruments and other resources. All such material and resources which help the teacher of mathematics in the realisation of his objective of effective teaching can be termed as aids in. teaching of mathematics. These aids are also named as audio-visual aids in the sense that they call upon the auditory and visual senses of students. The aids like radio, tape recorder which help the individuals to learn through listening, are called audio aids. The aids like filmstrip, projector, epidiascope, newspaper, magic lantern and black-board which help in learning through watching are called visual aids. Some aids like cinema, television, where one learns through listening as well as watching are known as audio visual aids.

Need and Importance of A.V. Aids in Teaching Mathematics

The use of audio-visual aids in teaching of mathematics may be supported on the following grounds:

1. Clarity of the subject:

+Audio-visual aids help in clarifying the various abstract concepts of mathematics instead of struggling hard only with the theoretical talks, if the teacher takes the help of some aid material he can make the' subject more clear and meaningful to his students. For example the simple facts of addition like 7+5 = 12 can only be taught effectively if the children are given opportunity to count seven and five concrete objects first separately and then in combination.

2. To make the subject interesting:

Audio-visual aids help in creating and maintaining interest in the learning of Mathematics. The subject no longer remains as boring, dull and unreal one.

3. Based on maxims of teaching:

The use of audio-visual aids facilitate to the teacher to follow the important maxims of teaching like, 'simple to complex', 'concrete to abstract', 'known to unknown' and 'learning by doing' etc.

4. Psychological value:

Use of audio-visual aids has some psychological advantage also. Children always like to manipulate or observe the new things. Once they are attracted towards an object or activity, their attention can be easily captured and desired interest in the learning can be safely maintained. The satisfaction of various interests and innate tendencies through audiovisual aids thus helps much in the task of learning.

5. Fixing up the knowledge:

The knowledge gained needs to be fixed in the minds of the students. It needs a lasting impression in their minds which can be easily engraved through audio-visual aids.

6. Saving of time and energy:

Much of the time and energy of both the teachers and the taught may be saved on account of the use of audio-visual aids as most of the abstract concepts may be easily clarified and understood through their use.

7. Use of maximum senses:

Senses are said to be the gateway of knowledge. Audio-visual aids help in the maximum utilisation of sense organs and there by facilitate the gaining of knowledge by the students.

8. Meeting the individual differences requirements:

There are wide individual differences among children: Some are ear minded, some can be helped through visual demonstration while others learn better through doing. The use of various types of audio-visual aids helps in meeting the retirements of different types of pupils.

9. Encouraging activity:

Teaching learning process becomes quite stimulating and active through audio-visual aids. Here passive listening does not help in the realisation of the objectives of teaching mathematics. Use of audio-visual aids helps in converting the passive environment of the class-rooms into a living one.

10. Development of scientific attitude:

Use of audio-visual aids helps in cultivating scientific attitude among students. Instead of agreeing to the listened facts, they resort to observe or use them practically with the help of audio-visual aids and ultimately adopt the habit of generalization through actual observations and experiments.

VARIOUS TYPES OF AIDS USED IN TEACHING OF MATHEMATICS

Mostly, the types of aids given ahead are used in the teaching of Mathematics.

1. Weighing and measuring instruments:

Various types of weighing and measuring instruments help in acquainting the students with different scales and units of weight and measurement. The teacher should try to take help of such instruments like tape measures, balances and weights, graduated cylinders etc. for gaining practical knowledge of the mathematical facts.

2. Drawing instruments:

Drawing instruments help much in learning mathematical facts and skills. Specially in geometry and mensuration the use of these instruments is a must. Therefore, every student of mathematics should be asked to keep a geometry box containing essential geometrical instruments and the teacher should try to take help of the wooden instruments for his demonstration work on the black-board.

3. Real objects:

As an aid real objects are said to be the most useful and effective means of providing direct experiences to the students. The list of such objects may consist of objects like beads, ball frames, coins, seeds, sticks, pebbles, coloured balls or solids, pencils, the material used and produced in various work- experiences areas etc. Various topics and concepts concerning four fundamental rules, average, percentage, fraction, profit and loss etc. may be successfully taught through these objects. The real life situations may also be exploited as an aid in the teaching of mathematics. Classroom may work as a real object for teaching area of the four walls. Similarly, black-board, classroom tables, play-grounds and gardening plots etc. may prove very helpful in teaching area and other facts of mensuration.

4. Models:

Models are the copies of the real objects. When for some reasons or the other it is not possible or advisable to use the real objects, models prove very useful and effective means of educating the students. As far as possible a model should be least expensive and be made by the students themselves. Models can be

successfully used to acquaint the students with the shape and forms of different numerals and geometrical figures. For this purpose even the square, round or rectangular process of cardboard or thick paper may serve as models. To give practice in writing numbers, numerals and digits engraved on the wooden or stone pieces may prove useful models. Also for teaching topics like, area of four walls, cross roads and other squares, rectangular and circular figures models may be made out of thick paper and cardboard.

Models may also serve the best purpose in teaching various concepts and facts related to geometrical theorems and exercises. For example to acquaint the students with the fact that 'sum of the three angles of a triangle is equal to two right angles'.

5. Pictures and Charts:

In case where it is not possible to have an appropriate model or use real objects, pictures and charts prove very useful aid in teaching Mathematics. For example to teach

fractions-simple, compound and decimal, fractional parts [rectangular or circular] may be drawn on the charts. In teaching profit and loss, unitary method, percentage, interest and work and time the help of charts may be taken to work out principles and formulae. Pictures may also be used to make the students understand the basic things about the problems. In the problems related to area, volume and mensuration, charts may be used for analysing the problems: In geometry, the use of charts may be made in showing figures concerning the proof of the theorem, or proposition and helping the students in the construction of various geometrical figures and diagrams. In algebra, the charts may be effectively used in teaching of directed numbers, four fundamental rules and problems based on equations. The general formulae may also be demonstrated through charts. The pictures and chart prove a helping hand to the teachers as they save their time, and energy otherwise wasted in drawing figures and diagrams on the black-board. In addition to this, they may prove constant source of inspiration and means of imparting self-education to the students. For this purpose the following types of charts and pictures may be hung in the classroom

i. Charts concerning geometrical figures and shapes.
ii. Charts depicting different principles and formulae.
iii. Charts concerning various units of weights and measures.
iv. Pictures concerning great mathematicians.
v. Pictures related to the history of mathematics.
vi. Pictures and charts showing use of mathematics in day to day life.

The following points should be kept in mind for the effective use of the pictures and charts :-

a. There should not be too many things or facts demonstrated through a simple chart. It should concentrate on a single definite purpose.
b. Charts should be coloured and attractive.
c. Charts should have a proper size. The drawing and writing must be so distinctive and clear that all the pupils may get benefitted through its use.
d. No irrelevant thing or theme not connected with the topic should be demonstrated through a chart.
e. As far as possible the charts and pictures should be got prepared by the students.

6. Black-Board:

As an aid in teaching of mathematics, black-board is so much effective that it is usually termed as the right hand of a mathematics teacher. The secret of the popularity of the black-board lies in the fact that one can write or remove what has been written on it at his own will without involving any significant expenditure. The teacher writes and explains his writing while writing on' the black board. In this way students get both the benefit of observing and listening at a time. The use of black-board is indispensable for all the branches and topics of Mathematics. It begins with the first lesson of mathematics and then goes up to highest learning in the subject. In all tasks like drawing of diagrams and figures, giving definitions, writing the language of the problem and their solutions, drawing generalizations and giving principles and formulae, having practice and drill work and assigning home-task, black-board proves a good helping hand. The black-board with graph lines may be successfully used for thawing all types of geometrical figures, plotting graphs, presenting statistical data, solving problems on areas and volume. In short, what a teacher wants to communicate to his students may be succesfully done through this readily available aid and it helps in quick understanding as well as fixing up of the knowledge of the subject. But for getting the desired advantages, the teacher must know its proper use. The following may serve a useful purpose in this direction :-

i. The black-board should be properly cleaned before making its use; it should be got polished or painted from time to time.

ii. The teacher should have sufficient practice for writing, sketching and drawing legibly on the black-board.

iii. The black-board writing should be in straight lines. It should be visible to all the students in the class.

iv. Teacher should try to speak what he is writing on the black board.

v. He must be very careful for not writing anything wrong and inappropriate on the black-board.

vi. He must learn how to face the black-board while writing on it. He should have a watchful eye over his students while keeping himself busy on the black-board.

vii. The problems solved or the work done on the black-board should follow a logical as well as psychological sequence.

viii. Teacher should always make use of the standard terminology and symbols on the, black-board so that the students may not get confused. ix. The chalk used should be of a superior quality. It should be properly pressed while writing.

x. Coloured chalks should also be used for emphasizing particular facts and making the diagrams and sketches more attractive and meaningful.

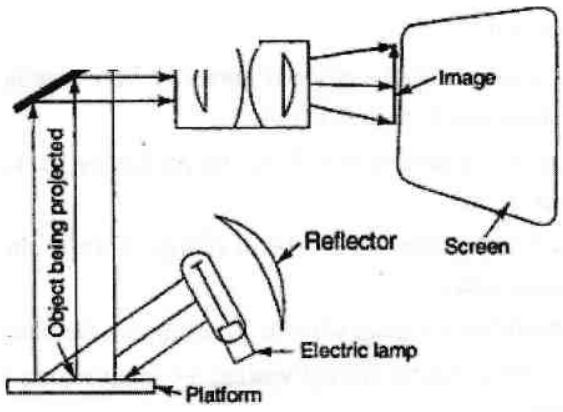
xi. The students should also get proper opportunity for solving the problems and drawing the diagrams on the black-board.

7. Magic Lantern:

This instrument of science had proved very useful for teaching mathematics. It helps the teacher to demonstrate different types of figures, diagrams, pictures related to various topics of mathematics through the slides. For getting better results, the teacher may also give explanation of the things demonstrated on the magic lantern. The demonstration may further be followed by group discussion for the clarification of the various issues on the topic.

8. Epidiascope:

This instrument is used for enlarging and then demonstrating the contents, figures and diagrams of the printed or handwritten pages. It has shown its value in teaching of mathematics too, specially at the time when the teacher feels difficulty in drawing or giving things on the blackboard.



Pretetten lens

9. Film-Strip Projection:

In a film strip, 15 to 20 slides concerning useful topics are photographed on a 35 or 16m.m. films. These film strips are then projected on the screen through a projector. The teacher may demonstrate the pictures for any period of time irrespective of speed as the situation demands. These film strips give altogether a new colour and attraction to different ideas in mathematics. The film strips may be easily obtained from the market or borrowed from the Central Library, Department of NCERT and SCERT and some leading foreign embassies.

10. Cinema:

Cinema, the popular, means of entertainment may be successfully used for the teaching of various principles, definitions and facts of mathematics. The life history of mathematicians, their discoveries and the historical landmarks of the development of mathematics may also be successfully demonstrated through cinema films. The films may be used effectively for teaching demonstrative geometry. The students may learn how to use various geometrical instruments, draw different types of figures and diagrams, survey and measure the different dimensions areas and volumes and use graphs. Though films students may be acquainted with the use of mathematics in solving day to day life problems used in different occupations and field of actions. In this way the use of cinema films may prove, quite effective, stimulating and useful for the teaching and learning of mathematics. Cinema is such an aid that calls on both the visual and auditory senses. The students at the same time may listen as well as observe the facts and therefore it provides to them a greater stimulating and motivating value for learning something new. It can help not only the students but also the teachers in acquainting them with the growing knowledge, and methods of teaching being adopted in other, countries. In a school hail or big room, the teacher may demonstrate films through some good projector. The films for this purpose may be borrowed from state or central education departments and libraries. The NCERT and similar state level bodies as well as some foreign embassies may also help in this direction.

11. Radio and Television:

Radio and television both have established their due credit in the field of education. Almost all the important centres of A.I.R. broadcast programmes concerning education. For the programmes on mathematics eduoatioi4 either the regular classes on topics of mathematics are being held or the important discussions and speeches concerning principles and laws of mathematics, life history and contributions of mathematicians, historical development of the knowledge of mathematics, the application of mathematics in practical life are broadcast. Highly experienced teachers, teacher educators, mathematicians and research scholars take

part in such programmes. The Radio as a means of communication takes their voices to the millions of students and teachers listening to their programmes. Television has far greater advantages as it not only conveys the voices but the pictures and actual scenes also. The students sitting far, away from the TV stations may be benefited through the telecasting programmes almost in the same way as it is happening just before their eyes. A teacher of mathematics should try to take advantage of such learning opportunities by making himself and his students fully conversant with such programmes.

12. Newspapers:

Newspapers may be used as an effective aid for teaching and learning of mathematics. They help in correlating teaching of mathematics with day to day- happenings of life. The statistics given in the newspaper in the form of weather charts, the prices of various commodities, budgets of state and central government, interest rates of various private and government agencies, stock and shares etc. all provide good means for making the teaching of mathematics interesting, useful and purposeful. The cutting

of the newspapers may thus be employed to help the students in learning the practical application of mathematics in day to day life.

13. Running cooperative store:

A cooperative store running in a school may also be utilized for mathematics education. The students may learn practically the various principles and facts of mathematics regarding profit and loss, four fundamental rules, unitary method, percentage, weighing and measuring etc. by becoming shopkeepers or customers in such a store.

14. Mathematical games and riddles:

Mathematical games and riddles besides playing their recreational roles, may be effectively utilised for learning, practising and using various principles and facts of mathematics. There are so many games which may be successfully utilized for mathematics education. In such games and riddles, competitions may be usually organised by dividing the students into groups. It will provide learning opportunities while playing.

15. Visits and excursions:

Visits and excursions do play an effective role in learning mathematics by providing . direct experiences. The students get opportunity of learning mathematics in the manner it is being used in practical life. For visits and excursions they may be taken to different places like workshops, industries, mills, and telegraph offices, station, market, agriculture fields, forest, some picnic spots and visiting places. During these visits students may face so many problems requiring the knowledge of mathematics for their solutions. The teacher may utilize

such situations for giving the necessary knowledge of mathematics or sometimes students may get opportunities for applying the already gained knowledge.

Conclusion about the Aids used

In this way it may be seen that there is no dearth of aids for making the learning of mathematics easy, interesting and useful. A resourceful teacher may choose the appropriate aids suiting to his needs, time and occasion. It is also true that financial resources put obstacle in the way of taking advantages from aids. But this is not the end of the story one should not get disappointed. If a teacher has the courage and will he may get these aids prepared with the help of his students- through indigenous and cheap material. In this direction the services of the agencies like education departments of central and state governments, teachers training institutions, NCERT and SCERT, and extension services centers may also be received for collecting the essential and useful teaching aids.

LOW COST IMPROVISED TEACHING AIDS

As emphasized earlier mathematics is a type of subject the need of which is felt well for performing the day to day activities of our life. Not only this a child from his tender age observes the use and application of mathematics in the existence and activities of all the animal and inanimate objects surrounding him. In fact nature itself is a big source and a treasurer of aid material for learning the facts and principles of mathematics. The children may thus be helped to gather valuable direct and indirect experiences for the learning of mathematics from their local environmental surroundings. In their local setup comprising of their homes, community, physical and social environment, they may get a lot of opportunity to practise and learn so many valuable concepts regarding the teaching learning of mathematics. There is a lot of cheap and sometimes waste material available in children's local environment that can be successfully utilised for the improvisation of valuable aid material for the teaching of mathematics.

1. Concrete material and objects:

A wide variety of collection of different types of concrete material like beads, seeds, balls, sticks, match boxes, pebbles, different types of corns etc. may be made with the help of students. All that material may prove quite helpful in the learning of counting, four fundamental rules [addition, subtraction, multiplication, division], multiplication tables etc.

2. Improvising an abacus:

An Abacus containing a number of wood, metal or even thermocol beads in several wires can be easily improvised for teaching the students the facts of counting, four fundamental rules, place value system, etc.

3. Place value pockets may also be improvised:

The required boxes for this purpose can be made with the help of thermocol, thick paper or wooden etc. and the system be made so operated as help in the learning of the concept of place value.

4. Preparation of models:

By using the easily available low cost or waste material various types of models may be improvised. Let its have different types of such models representing the shape of various geometrical figures like rectangle, square, parallelogram, trapezium, triangle, circle, ellipse, cone, cylinder, pyramid, sphere etc. We can have models made of clay, match box sticks, thermocol, wood, wax etc. to demonstrate the various geometrical figures, their properties and operations. Area of the cross roads, Area of the four walls of a room, circumference and area of a circle, circumference and area of rectangle, triangle and so many other things related to the learning of mensuration may be easily taught through the use of the models made of the locally available low cost material. The waste paper, card boards, thick papers, wooden boxes etc. available in the packages of the household goods purchased from market can be effectively used for making that model with a simple use of pins, nails, scissors, hammers, threads, ropes etc. easily available in the houses and chooi workshop. For illustration purpose let me point out a working model for helping the students to understand geometrical concept and the related theorem "sum of the three angles of a triangle is equal to two right angles. It has been already mentioned in the present chapter. We may utilise the wood available from the carton boxes or the boxes received from shopkeepers during the purchases of fruits and other household articles for the improvisation of this model. Match box sticks and rubber bands may also be utilized for the construction of such model.

5. Preparation of Charts and pictures:

With the help of the, chart paper and drawing material easily available in the market, the low cost visual aid material may be easily prepared for the teaching learning of almost all concepts related to all the branches of mathematics. You may hang these charts in the mathematics classrooms, laboratory or library for the grasping of the essential mathematical concepts with no extra efforts. At the time of class room teaching these charts and pictures may be successfully used for the clarity of the needed mathematical concepts, processes and operations.

A teacher of mathematics in this way may utilize the waste or last cost material available in the local surroundings for the preparation and improvising valuable teaching-learning aids for bringing efficiency and effectiveness to the ongoing teaching Learning process. He may even utilize the real surroundings as an aid to his teaching. For example while teaching about the area of the four walls of a room, he can have the class room as a living concrete model. In teaching the concept of mensuration, then he may utilise the sports ground, school, garden and neighbouring plots as a living models for necessary

surveying and measurement The scrap book prepared by the students through the news paper cuttings of the data of mathematical interest thus may also be used as one of the useful aid material for ac acquainting the students with the facts and principles of mathematics.

Paper folding and Paper cutting etc.

Paper folding encourages students toward content and invites then to reflect upon the meaning of proof. Significant benefit of paper folding technique is its accessibility to students and the effective benefits of learning.

Paper folding and paper cutting activities are selected because it is simple and interesting hands on activity that students can build their learning experience. The process incorporates both mental and physical involvement in the learning process. It is especially appealing to the concrete learner who need to visualize and sense to learn.

4.8 CRITCRIA FOR SELECTION OF APPROPRIATE TEACHING AIDS:

Teaching and learning material, whether purchased or donated, should be selected and acceded in was which ensure they:

- ♦ are directly related to a preschool's or school's curriculum policy and program, based on the department's framework of standards and accountability, and include, where relevant, support for the recreational needs of children and students
- ♦ support an inclusive curriculum, thus helping children and students to gain an awareness of our pluralistic society and the importance of respectful relations with others
- ♦ encourage understanding of the many important contributions made to our common Australian heritage by men, women, Aboriginal and Torres Strait Islander peoples, people from diverse cultural and linguistic groups, people with disabilities and minority groups
- ♦ motivate children, students and educators to examine their own attitudes and behaviour and to comprehend their duties, responsibilities, rights and privileges as citizens in our society
- ♦ are relevant for the age of the children or students for whom they are selected and for their emotional, intellectual, social and cultural development. This includes the

  assurance that children and students will not be exposed to offensive materials; that is, materials which describe, depict, express or otherwise deal with matters of nudity, sexual activity, sex, drug misuse or addiction, crime, cruelty, violence or revolting or abhorrent phenomena in a manner that a reasonable adult would generally regard as unsuitable for minors of the age of the relevant children and students
- ♦ provide opportunities for children and students to find, use, evaluate and present information and to develop the critical capacities to make discerning choicer so that they are prepared for exercising their freedom of access, with discrimination, as informed and skilled adults
- ♦ represent a range of views on all issues.

4.9 USE OF MASS MEDIA IN TEACHING MATHEMATICS

Let's take quick trip through the main types of media outlet. There are others webpages. CD-ROMs, DVDs, blogs, podcasting, whatever.

Magazines

Popular science magazines have the advantage that their readership is self-selected for an interest in

science. Surveys have shown that mathematics is very popular among such readers. Each magazine has its own level, and its own criteria for what will appeal. Scientific American is justly famous for the 'mathematical games' columns originated by the peerless Martin Gardner, which unfortunately no longer run. In the UK there are New Scientist and Focus, which regularly feature mathematical items ranging from primarily testing to su doku. If you are thinking of writing an article for such a magazine, it is always better to consult the editors as soon as you have a reasonably well formulated plan. They will be able to advise you on the best approach, and will know whether your topic 1640 Ian Stewart has already been covered by the magazine - a problem that can sink an otherwise marvellous idea. Expect the editors and subeditors to rewrite your material, sometimes heavily. They will generally consult you about the changes, and you can argue your case if you disagree, but you must be prepared to compromise. Despite this editorial input, the article will usually go out under your name alone. There is no way round this: that's how things are in journalism.

Newspapers

Few newspapers run regular features on mathematics, bar the odd puzzle column, but most 'quality' newspapers will run articles on something topical if it appeals to them. Be prepared to write 400 words on the Fields medallists with a four-hour deadline, though, if you aspire to appearing in the national news.

Books

Books, of course, occupy the other end of the deadline spectrum, typically taking a year or so to write and another year to appear in print. They really deserve an article in then-own right, and I won't say a lot about them here, except in Section 7 below. Sometimes expediency demands a quicker production schedule. I once wrote a book in 10 weeks. It was short, mind you: 40,000 words. The quality presumably did not suffer because it was short-listed for the science book prize. If you want to write semi-professionally, you will need an agent to negotiate contracts. At that level, book writing is much like getting a research grant. Instead of ploughing ahead with the book, you write a proposal and go for a contract with a specified advance on royalties.

Radio

Radio is my favourite medium for popularising mathematics. This is paradoxical, because radio seems, to have all of the disadvantages [such as no pictures] and none of the advantages [such as being able to write things down and leave them in full view while you discuss them] of other media. However, it has two huge advantages: attention-span and imagination. Radio listeners [to some types of programme] are used to following a discussion for 30 minutes or longer, and they are used to encountering unfamiliar terminology. And radio has the best pictures, because each viewer constructs a mental image that suits them. On radio you can say 'imagine a seven-dimensional analogue of a sphere" and they will. It may not be a good image, but they'll be happy anyway. Say the same on TV and the producer will insist that you build one in the studio for the viewers to see. TV removes choice: what you get is what they choose to show you. On radio, what you see is what you choose to imagine.

Television

Television is far from ideal as a medium for disseminating science, and seems to be becoming worse. As evidence: every year the Association of British Science Writers presents awards for science journalism in seven categories in 2005 no award was made in the television category, and [Acker] the judges

stated: Mathematics, the media, and the public 1641. To say the quality of entrants was disappointing is an understatement. We were presented with 'science' programmes with virtually no science in them. Some were appalling in their failure to get across any facts or understanding. Whenever there was the possibility of unpicking a little, highly relevant, science, or research methodology, the programmes ran away to non-science territory as fast as possible, missing the whole point of the SLOP, as far as we were concerned. I still vividly recall a TV science programme which informed viewers that Doppler radar uses sound waves to observe the speed of air in a tornado. No, it uses electromagnetic waves - the word 'radar' provides a subtle clue here. Sound waves come into the tale because that's where Doppler noticed them. The reasons why television is far from ideal as a medium for disseminating are equally disappointing. It is not the medium as such that is responsible - although it does discourage attention-spans longer than microseconds. The responsibility largely rests with the officials who commission television programmes, and the companies who make them. Television changed dramatically in the 1980s, especially in the UK. Previously, most programmes were made "in-house' by producers and technicians with established track records and experience. Within a very short period, nearly all programming was subcontracted out to small companies [many of them set up by those same producers and technicians] on a contract-by-contract basis. This saved television companies the expense of pensions schemes for their employees [since they now had none] and protected them against their legal responsibilities as employers [ditto]. But as time passed, contracts were increasingly awarded solely on the basis of cost. A new company would get the commission to make a programme, even if they had no experience in the area, merely because they were cheaper. Very quickly, most of the companies that knew how to make good science programmes were ousted by new kids on the block whose main qualifications were degrees in media studies and, the decisive factor, cheapness. Any lessons previously learned about how to present science on television were lost, and had to be re-learned, over and over again, by a system dedicated to the perpetual reinvention of the wheel. There is still some good IV science, but nowhere near as much as there ought to be given the proliferation of satellite and cable channels. The good news here is that TV is once again wide open as a medium for popular science, especially now that there arc hundreds of channels desperate for content. But we will have to fight all the old battles again.

4.10 FIELD TRIP AS A TEACHING TECHNIQUE

- A field trip is defined as any teaching and learning excursion outside of the classroom.
- There are two types of field trips - Physical and Virtual.

Physical Field Trip Examples

- school playground, school board outdoor education centres, provincial parks, protective wetlands, science centres, museums, zoos, grocery stores, fire stations, veterinary clinics, agricultural operations, natural resource operations

Why Field Trips?

- To make a connection between reality and theory - hands-on
- Can be used as an introduction to a unit or a culminating actively.
- To provide an authentic learning experience
- Exciting, children get to meet and interact with others
- They can experience all five senses, see, touch, feel, smell, taste

- Children remember the field trips because they learn using different methodology **How to Plan and Run a Successful Physical Field Trip**

- Planned and effectively organized
    - o Check for school/board policy on field trips o Children to supervisor ratios o Transportation procedures o Fund raising
- Plan with children as much as possible
- Involve school principal and vice-principal
- Ensure field trip compliments the curriculum by meeting specific expectations

- Ensure students have necessary background knowledge prior to field trip, if introduction to field trip provide essential preparatory information in order to prepare students for the experience
- Plan post-trip activities that build on the knowledge gained in partaking in the field trip [eg. reports, displays, photos, graphs].
- Prepare a checklist to ensure that all tasks are completed [e.g. booking facilities and transportation, parental notifications, medical forms, supervision, safety precautions, emergency information] and have the school administrator sign the checklist once completed.
- Be sure to visit the site ahead of time, in order to plan for safety, resources and resource personnel, facility.
- Plan on route activities to enrich their experience during the field trip.
- Provide parents with rationalization for the field trip and trip itinerary.

Guidelines for Safety and Behaviour

- There are many potential liability situations that can occur on a field trip, it is your ultimate responsibility to ensure that the following safety guidelines are meet concerning safety and behaviour while outside the classroom.
- Set behavioural expectations for the field trip and describe and discuss them with the children prior to departure.

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- Have children create their own code of behavi our with teacher involvement and veto power.
- If junior students are mature enough to be responsible and accountable for their own behaviour, have them sign a written code of conduct; therefore, creating a behavioural contract.
- Introduce the idea of team work to enable students to live to the written code of conduct.
- Describe the consequences for not behaving properly prior to embarking on the trip.
- Provide parents with behavioural expectations and ask them to ensure that the children know and understand the code of conduct and the consequences.
- Ensure that the student/supervision ratio meets board/school standards.
- Eliminate all the safety concerns identified in the school/board policy.
- Use board approved transportation.

- Create passenger manifest and file with appropriate school personnel. Also, take along passenger manifest to check that everyone is accounted for.
- Implement a buddy with students as an additional safety precaution.
- Ensure that safety gear and first aid equipment are readily available and in plain view.

Tips and **Variations for Field Trips**

Before the Trip, Teachers Should:

- Visit the site to find connections to curricula, assess potential problems, and plan how the students could best use their time.
- Give as much context as possible bso that the students will understand what they see. Teachers might consider having the students do something like a journal or a K/ W/L chart in which they list questions they have, expectations for their visit, or plans for ways to use what they will see.
- Create a trip sheet like Stanlee Brimberg's that prompts students to draw, write responses, answer questions, or find items for a "scavenger hunt" of the location. This sheet, however, should not be so directive that the students can't see and respond to the site in their own ways.
- Set standards of etiquette and respectful behavior.

During the Trip, Teachers Should:

- Build in opportunities for students to view the site or work alone, in pairs, or in small groups. On a trip to a museum, for example, the students could be asked an open-ended question like, "Find a work that represents our theme or time period and sketch it. In class we will share our choices and discuss why we chose them." The students could also choose one aspect or part of the site to explore.
- Consider giving some students disposable cameras, small tape recorders, or mandates to record specific information. When the class is back at school, they can compile a complete picture.

After the Trip:

- **J*** Allow the students to synthesize their experience creatively. For example, they might create trip brochures for other classes or the school library. They might create children's books about a theme from the field trip. Or they might present their experience orally to another class or grade.

Benefits of Field Trips

- ♦♦♦ Field trips bring classroom study alive for students and help them remember and relate to what they have learned. They provide rich resources that can rarely be approximated in the classroom. They also help connect school to the world.
- Field trips provide new cultural contexts for literature and provoke questions.
- Field trips stimulate and focus class work by helping students synthesize information.

4.11 QUALITIES OF GOOD MATHEMATICS TEXT BOOK

A] Introduction

The mathematics textbook is an important source for learning mathematics and it plays a key role in effective teaching and learning. A textbook should stimulate reflective thinking and develop problem-

solving ability among students. The textbooks should present real learning situations, which are challenging and interesting for the students and should not render itself as a means of rote learning.

Text books and teachers' guides occupy a unique place in the teaching learning process. Text book are an indispensable part of primary and secondary education. The text book is a teaching instrument. It is not only a source of information, but a course of study, a set of unit plans and learning guide. It helps to revise and reinforce the language material already taught. In the absence of any other instructional material, the text book becomes a potent tool in the hand of a teacher to teach the skill of a language and the more so of a foreign language.

B] Qualities of a Mathematics Textbook

The qualities of a good textbook in mathematics can be broadly classified under the following heads

1] Physical features
2] Author
3] Content
4] Organization and presentations
5] Language
6] Exercise and illustration
7] General

1. Physical features:

- Paper: the paper used in the textbook should be of superior quality

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- Binding: it should have quality strong and durable binding
- Printing: it should have quality printing, bold font and easily readable font.
- Size: bulky and thick. It should be handy
- Cover: it should have an appealing and attractive cover page.

2. Author:

- Qualified author should write it
- Experienced teacher should write it
- Competent teachers should write it
- It should be written by committee of experts constituted by the state government
- For the authors, certain minimum academic and professional qualifications may be prescribed.

3. Content

- It should be chiId centered
- The subject matter should be arranged from simple to complex and concrete to abstracts.
- The subject matter should create interest in the pupil.
- It should be obj ective oriented
- It should be written according to prescribed syllabus
- It should satisfy the demands of examination

- The answers given at the end of each section should be correct
- It should include the recent developments in the mathematics relating to the content dealt with.
- Oral mathematics should fine its due place in the textbook.

4. Organization and presentation
   - It should provide for individual differences.
   - There should be sufficient provision for revision, practice and review.
   - It should stimulate the initiative and originality of the students
   - It should offer suggestion to improve study habits.
   - It should facilitate the use of analytic, synthetic, inductive, deductive, problem solving and heuristic approaches to teaching.
   - Content should be organize in a psychological consideration
   - Content should be organize in a logical way
   - It should suggesting project work, fieldwork and laboratory work.
5. Language
   - The language used in the textbook should be simple and easily understandable and within the grasp of the pupils
   - The style and vocabulary used should be suitable to the age group of student for whom the book is written.
   - The term and symbols used must be those, which are popular and internationally accepted
   - It should be written in lucid, simple, precise and scientific language.
6. Exercise and Illustrations:
   - The illustrations should be accurate
   - The illustrations should be clear and appropriate
   - It should contain some difficult problems
   - It should contain exercises to challenge the mathematically gifted students.
   - There should be well-graded exercises given at the end of every topic.
   - The exercise should develop thinking and reasoning power of the pupils.

7. General:

- At the end of book there should be tables and appendices.
- The textbook should be of latest edition with necessary modifications
- The book should be moderate price and readily available in the marker.

C] Conclusion

Every teacher of mathematics uses a textbook. An average teacher uses it as "his stock in hand" but a good teacher uses it as "a helper". The textbook that is considered "as store house of basics information" can facilitate a teacher to do wonder in his subject:

The textbook should not be used as the only source of instructional material. It should be used as an aid in teaching. **The following are the qualities of a good math textbook:**

I] A mathematics textbook should be written in accordance with the aims and ob jectives of teaching the subject in that particular class.

2] It should be well illustrated.

3] There should be diagram and figures wherever needed.

4] The textbook should be written in simple and understandable language.

5] It should be free from mistakes.

6] It should be written within the grasp of the children.

7] It should provide sufficient materials to motivate the students to solve problems.

8] The students should get adequate opportunity of learning through initiative and independent efforts.

9] The problems should relate to the real life needs and physical & social environments of the learners.

10] It should foster the right attitude towards self-study and self-reliance among pupils and it should be done by promoting project works, field works and laboratory works.

II] It should promote the use of analytic, synthetic, inductive-deductive, problem-solving and heuristic approaches to teaching.

12] The content should be up-to-date.

13] The exercises should aim of all level of students. It should be challenging for intelligent students and should give opportunity for average & below average students also.

14] It should satisfy individual difference in students and should meet the varying abilities, interest and attitudes.

15] It should promote logical and psychological arrangement of contents.

16] It should provide for practice, revision and satisfy the demands of examination.

17] Also the textbook should be appealing and should have the necessary external qualities i.e., its get-up, paper and printing, etc., should be good.

EXERCISE

1. Which is the best method of teaching mathematics according to your option? Support your preference with arguments.
2. Illustrate and discuss the inductive-deductive methods of teaching mathematics.
3. Heuristic spirit is the golden rule which a teacher of mathematics should never forget, whatever method of teaching he may adopt?
4. How will you employ the project method for the teaching of mathematics?
5. What do you understand by problem method? What is its scope in the teaching of mathematics? How will you employ it?
6. What do you understand by the term "teaching aids"? Discuss their need and importance.
7. Discuss the role of teaching aids in the teaching of mathematics of the school stage.
8. Give your views regarding the development of low cost improvised teaching aids in the subject mathematics.
9. Write the advantages of lecture method.
10. Write the disadvantages of lecture method.
11. Write the purpose of demonstration method.
12. Write the criteria for a good demonstration method.

Unit V EVALUATION AND ANALYSIS OF TEST SCORES

5.1 Different types of tests in Mathematics, achievement, diagnostic, prognostic

5.2 criterion and norm referenced evaluation

- Construction of achievement test
- Continuous and comprehensive evaluation
- Formative and summative assessment
- Grading pattern

5.3 Statistical measures

- Mean, median, mode, range, average deviation, quartile deviation, Standard deviation
- rank correlation
- Graphical representation of data
- Bar diagram, Pie diagram, Histogram, Frequency Polygon, Frequency Curve and Ogive curve.

INTRODUCTION

measurement is an important feature of our daily life. From birth to death, almost every aspect of our life is touched by measurements in its natural form. Measurement enters into all branches of science. Measurement is an effective in physical sciences, biological sciences, social sciences as in applied sciences. For that reason measurement is indispensable in the domain of education.

What is Measurement?

Measurement of any kind is determining how much or how little, how great or how small, how much more than or how much less than. In the words of James. M. Brad field "Measurement is the process of assigning symbols to dimensions to phenomena inorder to characterize the status of a phenomena as precisely as possible". However measurement in education is more complex than measurement in physical situation. Educational measurement involves the mental processes of the individual which are not visible and which are interpreted in terms of the behaviour of the individual in certain situations.

What is Examination?

An educational examination may be defined as the assessment of a persons performance, when confronted with a series of questions, problems or tasks set in order to ascertain the amount of knowledge that he has acquired and the extent he is able to utilize or the quality and effectiveness of the skills he has developed. The examination is intended only to focus at the tangible and easily measurable objectives of education like knowledgeand skill. As an increasing emphasis was given in educational philosophy to other goals of educational effort satisfaction mounted with the existing system of examinations. Attempts were accordingly made to release examinations and to replace by a wider and more encompassing

concept of evaluation.

Evaluation:

evaluation in general is an act or a process that allows one to make a judgment about the desirability or value of a measure, evaluation in educational situation is thus a relatively new term introduced to designate a more comprehensive concept of measurement than is implied in conventional tests and examinations, the emphasis in evaluation being upon broad personality changes and more objectives of an educational programme and therefore include not only subject matter achievements, nit also attitudes, ideals, ways of thinking, work habits and personal and social adaptability. Thus evaluation is not just a testing programme. Tests are but one of the many different techniques that may contribute to the total evaluation programme. Evaluation is any systematic continuous process of determining

- the extent to which specified educational obj ectives previously identified and defined are attained.
- the effectiveness of the learning experiences provided in the class room.
- how well the goals of education have been accomplished.

Thus evaluation is integrated with the whole task of education and its purpose is to improve instruction and not merely to measure its achievement. In its highest form evaluation brings out the factors that are inherent in student's growth such as proper attitudes, habits, manipulative skills, appreciations and understanding in addition to the conventional acquisition ofknowledge.

evaluation is the process of finding out the extent to which the desired changes in behaviour have taken place in the student. It differs form the concept of measurement in the sense that evaluation is more comprehensive. Measurement consists of rules for assigning number to attributes or characteristics of behaviour where as the evaluation aims at providing detailed and comprehensive meaning and interpretation to the bahavioural attributes of a learner. It expresses quantitative as well as qualitative description of learners' performance. Te purpose of evaluation is different at various stages of instruction. Prior to beginning of instruction, the assessment of the learners' present achievement should serve the basis for selecting and formulating instructional objectives and then for planning appropriate learning experiences. The evaluation also helps the teacher to know how effective the instruction has been in helping learners to master the "instructional objectives"

Effective evaluation of student's achievement with respect to accepted and planned objectives of instruction is considered an indispensable aspect of good teaching. Teachers use various evaluation procedures i.e., tests (oral and written), practical, assignments, observation, interview etc., for assessing and monitoring the progress of the students achievement in scholastic and co -scholastic areas. These evaluation procedures and techniques have become an integral part of the instructional process and influence students in many ways. One of the functions that evaluation serves is to enable students to determine how well they are learning and achieving. When students are aware of the learning progress, their performances will be superior to what it would have been without such knowledge. The **purpose** of **evaluation**

- To provide information for grading, reporting to parents and promoting students.
- To evaluate the effectiveness of a single teaching method or to appraise the relative worth of several methods.
- To motivate the students.
- To select students.

- To evaluate the entire educational institution and to show how several of its aspects could be improved.
- To collect information for effective educational and vocational counseling.

5.1 TYPES OF TESTS IN MATHEMATICS:

- Achievement test
- Diagnostic Test
- Prognostic test
- Criterion test
- Norm referenced test

Achievement test:

The term achievement is often understood in terms of pupil's scores on a certain school test. If, for instance, a student is tested in two school subjects, say Language and Arithmetic and in one subject he gets 70% marks while in the other 60% marks, it is understood that his achievement in language in which he gets 70% marks is better that that in arithmetic in which he gets 60%. This is a loose way of understanding the concept of achievement. More intelligently understood, achievement means ones learning attainments, accomplishments, proficiencies, etc., achievement is directly related to pupils growth and development in educational situations where learning and teaching are intended to go on.

The concept of achievement involves the interaction of three factors, namely, aptitude for learning, readiness for learning and opportunity for learning. Besides these factors, the concept involves health and physical fitness, motivation and special aptitude, emotional balances and unbalances. Achievement in education, precisely speaking, implies ones knowledge, understanding or skills in a specified subject or a group of subjects.

Achievement test constitute an important tool in school evaluation programme. It is necessary for the teacher to know how far the pupils have attained in a particular subject-area. Pupils differ in their attainments. In the school evaluation programme, various forms of achievement tests are used to measure the extent of learning of the pupils.

"Any test that measures the attainments or accomplishments of an individual after a period of training or learning is called an achievement." - N.M. Dowinie.

"An achievement or proficiency test is used to ascertain what and how much has been learnt or how well a task can be performed, the focus is on evaluation of the past without reference to the future, except for the implicit assumption that acquired skills and knowledge will be useful in their own right in the future."- Super.

"Achievement tests are useful aids for diagnosing a students specific learning needs, for identifying his relative strengths and weaknesses , for studying his progress and for predicting his success in a particular curriculum".- Waters.

Of all the different types of examinations, achievement tests are used most frequently. Achievement tests differ from intelligence or aptitude tests in that - former measures the quality and quantity of learning attained in a subject of study or group of subjects after a certain period of instruction, the later measure pupil's innate capacity for attainment or accomplishment independent of any learning. These tests predict performance in a certain subject or group of subjects.

Functions of achievement tests

The major functions of achievement tests are:

- To provide basis for promotion to the next grade.
- To find out at the beginning of a year where each student stands in the various academic areas.
- In many schools, a certain grade or class has a number of sections. Achievement tests help in determining the placement of a student in a particular section.
- A teacher can use achievement test to see for himself how effectively he is doing, what is getting across pupils and what is not.
- To motivate students before a new assignment is taken up.
- Achievement tests expose pupils difficulties which the teacher can help them solve.
- To report to the parents the place of a student in a particular section according to the achievement scores.
- To diagnose a students specific learning needs, relative strengths and weaknesses.
- To predict future progress and for predicting his success in a particularly curriculum.
- To reflect teachers effectiveness.

Advantages to the teachers and administrators:

- Through achievement tests, the teacher can know the general range of abilities of students in the class.
- The teacher can select appropriate materials of instruction.
- The teacher can determine and diagnose the strengths and weaknesses of students in various subjects.
- The teacher can find out gifted and backward children.
- Tests help to discover backward children who need help and plan for remedial instruction for such students.
- Tests help to select talented pupils for providing enhanced curriculum.
- Through test, teachers can select students for the award of special merits or scholarships.
- Tests help to evaluate the extent to which the objectives of education are being achieved.
- They help to discover the type of learning experiences that will achieve those objectives with the best possible results.

Types of achievement tests

Achievement tests are of two main types:

- Teacher-made achievement tests and
- Standardized achievement tests.

Teacher made achievement tests may further be divided into two categories.

1. Written or paper and pencil test
2. Oral tests

Written tests can still further be classified as: (i) Long essay type (ii) Short answer type (iii) Objective type

Teacher-made achievement tests

Oral tests

In the past, classroom teachers relied very heavily on the oral work of pupil's in order to arrive an estimate of the extent to which they measured the work of his course. The value of the oral examination is quite apparent. Oral tests are the oldest form of achievement tests. These tests are mostly used in lower classes. But, even in higher classes, oral tests or examinations are used. The viva voce as used in graduate and post-graduate classes is nothing but oral examination. An advantage of this test is that a large number of areas can be covered and knowledge of the student can be assessed. Another advantage is that both the examiner and the examinee sit face to face with each other and the examiner can give a proper turn to the test, as the situation demands. More over, it is less time consuming. However, the chief limitations of oral tests are:

i. It is difficult to test each pupil on the basis of total curriculum
ii. It is difficult to ask same questions from every pupil.
iii. The examiner does not have any written proof regarding the pupil's attainment.
iv. Much depends upon the examiners personal choice.

Short Answer Type Test

Depending upon the length of the answer we can have short answer and long answer questions. A question which can be answered in less than tour steps may be called a short answer question. For example the following questions will be treated as short answer ones.

1. Prove that log(l+2+3) = logl+log2+log3
2. For what value of K will $25x^2$ +70x+K be a perfect square
3. In a right angle triangle ABC, AB=4cm, AC=3cm, what is the measure of the hypotenuse BC?

This form of questions can be easily related to objectives and can be made more stimulating for the pupils. The responses will be more specific than the long answer type.

Objective Type Questions

The essay type questions require the formulation of an extended verbal answer to the question. Objective type tests on the other hand consists of questions to which the pupil responds by the selection of one or more of several given alternative, by giving or filling in a word of a phrase or by some other device which does not call for an extensive written response.

FORMS OF OBJECTIVE TYPE TEST ITEMS

Simple recall tests

In this type of test in which the pupil is required to recall a response from his past experience to a direct question, a specific direction or a stimulus word or phrase. It completely eliminates the elements of guessing.

Examples

1. How many apples can I buy for rupees 50 if each apple cost Rs.10?--------

2. What is he formulae for the circumference of a circle?--------

Completion Type (Supply Type)

A competition test comprises a series of sentences in which certain important words or phrases have been omitted and blanks are supplied for the pupils to fill in. There may be more than one blank and each may be taken to count one point. This test, if carefully prepared has a wide applicability; it is likely to measure rote memory. The scoring of competition test is a laborious job, as the responses are scattered all over the page.

The competition item requires he pupil to complete the sentence by filling in the word or words that have been omitted or if directs him to respond to a question by writing the answer in the blank space provided. Because the pupil needs to decide upon his answer and then write it out. Test composed of competition items takes longer time to administer them than other forms of objective tests.

Examples:

1. The degree of the polynomial 2x2 y+3x+2 is----------

   What is the degree of the polynomial? ----------
2. The formula for the circumference of a circle is----------
3. Area of a square is----------
4. Perimeter of a rectangle is --------

The competition item, however offers a natural form of questioning. It can be used readily with material calling for specific information. These items are particularly useful for using mathematics and science where the results of complex reasoning processes can be represented by or few symbols or numbers.

Suggestions for Writing Completion Type Items

- ♦ If possible, use a direct question, rather than the complicated declarative sentences.
- ♦ The blank must call for a single specific response of the question can be answered by a unique word, number or symbol.
- ♦ Avoid using statements lifted directly out of the book, since this tends to over emphasize rote learning.
- ♦♦♦ In computational problem specify the units in which the answer is to be given and also the degree of precision expected.
- ♦ Avoid indefinite statements.
- ♦♦♦ Avoid lifting statements straight form the text book. This brings into prominence rote memory.
- ♦ Blanks should be of uniform length.
- ♦ Statements should be so chosen or worded that there is only one correct response to be supplied in the blank.
- ♦ Avoid grammatical clues to the answer expected.

For example

1. Give the value of 5 correct to 3 decimal places.
2. What is the area of the cloth in square meters needed for a conical tent of height 4m, radius of whose base is 175 cm?

Alternate Response or True-false Items.

An alternate response test is made up of items each of which admits only two possible responses. The usual form is the familiar true-false, right-wrong, correct-incorrect, yes-no test. It is adapted to the testing of simple facts ideas and concepts. Scoring is easy and objective. True -false item requires the pupil to express his judgment of a given statement by indicating true or false, yes or no, correct or incorrect, right or wrong or some similar response.

True false item should only be used when a simple statement is either completely true or completely false. Since only a small percentage of important items in most areas of

learning meet this criterion, the number of true false items which can be asked is limited. True false item is a special type of multiple choice question having only two choices.

Example

1. The quantity denoted by 11,800 can also be written as 11x103 (T/F)
2. If the base angles of a triangle are equal, the sides opposite to them are also equal (T/F)

In Constructing True-false items should be taken to avoid Several Common Pit Falls

1. Avoid broad generalizations
2. Use true or false item form only for statement which are either absolutely true or false, there should no exception.

Example

One of the co-ordinates in any point on x axis is zero (T/F) Here the exception

of origin may create confusion. A better way of asking the question would have been

At least one of the co-ordinates of any point on x-axis is zero.

3. Avoid giving clues in the statements

Words like all, always, never, only, often some times, generally, may etc., will give clues in answering them.

Example

An angle may be equal to its complement.

Here the word "may" is likely to give a clue to a clever student.

The question could better be written as

There exists an angle which is equal to its complement (T/F)

4. The use of negative statements in true-false items should preferably be avoided.

Example

If the base angles of a triangle are equal, the sides opposite to them cannot be unequal.

It could be simply worded as

If the base angles of a triangle are equal, the sides opposite to them are also equal.

5. Every statement involved in a true-false item should be self contained.

Example

The radius of a circle whose area is 75 would be greater than 5.

The question seems to be making very bold assumptions about the units of measurement involved in it.

The ambiguity could be removed by wording it as follows.

The radius of a circle whose area is 75 sq. cm would be greater than 5 cm. (T/F)

6. Avoid double statements each true-false items should test a single concept.
7. Avoid lifting up statements from the text books.
8. Have the number of true statements equal to the number of false statements.
9. Omit specific determiners that are likely to be associated with a true or false statement.
10. Avoid ambiguous statements or statements which may be correct by one interpretation and wrong by another interpretation.
11. Avoid statements in complex structure or figurative language.
12. Require the simplest method of response.

Multiple Choice Items

The multiple choice item consists of a stem, which presents a problems situation and several alternatives, which provide possible solutions to the problem. The stem, may be a question, or an incomplete statements. The alternative includes the correct answer and several plausible wrong answers called distracters. Their function is to distract those students who are uncertain of the answer. This test is made up of a number of items each which carries two or more responses out of which only one is correct or definitely better.

The following items illustrate the use of both the question form and the incomplete statement form of multiple choice items.

Example

1. Which one of the following is an example of a quadratic expression?
   a. X+2
   b. X+Y=4
   c. X^2+2x+3=0
2. An example of a quadratic expression is
   a. Y-t-0
   b.X+Y=4
   c.X^2+2X+3=0
   d.YH4Y+6
3. An example of a quadratic expression is

a.X+2 b.X+Y=4 c. X^2+2X+3=0 d.Y^2+4Y+6

4. Correlation lies between

a. 0&+1 b. -1 &+1 c. -1&0 d. -2 &+2

The alternative in the above examples certainly only one correct answer and the distracters are clearly incorrect. Multiple choice items typically include either four or five choices. The larger number will of course reduce the student's chances of obtaining the correct answer by guessing.

Rules for Constructing Multiple Choice Items

- ♦ Design each item to measure an important learning outcome.
- ♦ Present a single clearly formulated problem in the stem of the item.
- ♦ State the stem of the item in simple, clear language.
- ♦ Put as much of the wording as possible in the stem of the item.
- ♦ Avoid repeating the same material over again in each of the alternatives.

For example

Hexagon is defined as

a. Polygon with four sides
b. Polygon with five sides
c. Polygon with six sides
d. Polygon with seven sides

This could more elegantly be put as a hexagon is a polygon with number of sides equal to

a. 4 b. 5 c. 6 d. 7

- ♦ State the stem of the item in positive form whenever possible.
- ♦ Emphasis negative wording whenever it is used in the stem of an item.
- ♦ When negative wording is used in the stem of an item, it should be emphasized by underlining or capital letters or by being placed near the end of the statement.

Example

All of the following are quadrilaterals except

a. Rectangle
b. Square
c. Parallelogram
d. hexagon

- ♦ Make sure that the intended answer is correct and clearly best.
- ♦ Make all alternative grammatically consistent with the stem of the item and parallel in form.
- ♦ $he correct answer is usually carefully phrased so that it is grammatically consistent with the stem. A general step that can be taken to prevent grammatical inconsistency is to avoid using the article 'a' or 'an' at the end of the stem.

Example

In the expression 3x, x would be called as

a. Root

b. Base
c. Radical
d. Exponent

Here the right answer is the only alternative that correctly follows the article 'an' One could easily modify the question as In the expression 3x, x would be called

a. a root
b. a base
c. a radical
d. an exponent

Avoid verbal clues which might enable students to select the correct answer or to eliminate an incorrect alternative.

Some of the verbal clues commonly found in items are

a. Similarly of wording in both the stem and the correct answer.
b. Stating the correct answer in text book language.
c. Stating the correct answer in greater detail.
d. Including two responses that have the same meaning.

- Make the distracters plausible and attractive.
- Vary the relative length of the correct answer to eliminate length as a clue.
- Avoid use of the alternative 'all the above' and use 'none of the above with extreme caution.
- Vary the position of the correct answer in a random manner.

Matching type items

A matching type typically consists of two columns, each tern in the first column to be paired with a word or phrase in the second column upon some basis suggested. Usually, the number of items in the two columns is exactly the same. In the words of Lindquist, "The matching exercise is particularly well adapted to testing in who, what, why and when types of situations, or for naming and identifying abilities." In fact, matching comprises an economical form of having a number of multiple-choice items in the same question.

The matching type is simply a modification of the multiple choice form. Instead of testing the possible responses underneath each individual stem, a series of stems, called premises is tested in one column and the responses are tested in another.

Rules for Constructing Matching Items

- Include only homogeneous material in each matching item.

  As far as premises and responses are concerned they should be homogeneous with in a single item. They should be related to the same concept more or less. If for example they deal partly with algebra and quadrilateral, the students will get a number of clues in the item and it will serve no purpose.

- Keep the lists of items short and put the brief responses on the right.

  The list of premises and responses should be relatively small. It is difficult to provide long lists and to maintain homogeneity at the same time.

- Use a larger or smaller number of responses than premises and permit the responses to

be used more than once.

The two columns should seldom be the same length so as to provide perfect one-to-one matching. Preferably the number of responses should exceed the number of premises by two or three. This is done to control the chances of guessing and having the test answer by the elimination itself. He can attain the same effect by permitting some of the responses to be used more than once in the same time.

♦ Specify in direction the basis for matching and indicate that each response may be used once, more than once or not at all.

The basis on which the matching is to be done should always be made clear, if it is not obvious. This can be done by making the directions clear and specific and also by providing appropriate column headings.

Example

Consider the following matching question which does not follow some of the rules stated above.

Directions: Match column I with Column II by writing appropriate numbers in the blank spaces.

	COLUMN 1		COLUMN II
1	Isosceles triangle	A	Diagonals neither bisect nor are at right angles
2	■are	B	is bisect but are not at right angles
3	Rhombus	C	Dia>.;. e at right angles but only one is bisected by other
4	Quadrilateral	D	Diagonals bisect at right angles but i ;
5	Parallelogram	E	All the four sides and for angle s are equal
6	Kite	F	Two sides arc equal
■:: ■'/ .	Equilateral triangle	G	All t ies are equal

One can easily see a number of defects in the items. At the out set, the directions are two sketchy to clarify the basis of matching to the students. The number of premises and responses is the same to provide one-to-one matching. The homogeneity is test by mixing together the properties of triangle and quadrilateral.

Limitations of Matching Tests

♦ It is not well adapted to the measurement of understanding as distinguished form mere memory.

♦ With the exception of the true-false test, the matching test is form most likely to include irrelevant clues to the correct response.

♦♦♦ Unless skillfully made, it is time-consuming for the pupils.

How **to** improve Matching Test Items:

♦ Include only homogeneous or related materials in each matching type exercise. Dissimilar matches such as persons and events, dates and events, terms and definitions should not

be mixed together.

♦ Indicate clearly the basis for the matching. ♦♦♦ Avoid making the test too easy.

♦ Be sure that exercises do not indicate clues to matching pairs.

♦ The complete matching exercise should appear on the same sheet and be not carried over to the next pages.

♦ The matching test should contain at least five and not more than fifteen items.

♦ The items in the response column should be arranged in a systematic order.

General considerations for Writing Objective Type Tests

♦ Rules governing good language expression should be observed.

♦ Difficult words should be avoided.

♦ Text book wording should be avoided.

♦ Ambiguities should be avoided.

♦ Items having obvious answers should not be used.

♦ Clues and suggestions as far as possible should be avoided.

♦ Items that can be answered by intelligence only should not be included. ♦♦♦ Quantitative words rather than qualitative one should be used.

♦ Catch-words should not be employed.

♦ Items should not be inter-related.

♦ Response positions should preferably be aligned.

Advantages of Objective Type Tests

♦ Objective type tests have the major advantage of extensive sampling. A test which contains a hundred or more items, will adequately sample pupil achievement for many purposes.

♦♦♦ Objective test items are so stated that usually the answers to them are brief and only one correct answer is possible for a certain item. There should be no disagreement between different persons scoring the items at the same item of different examinees.

♦♦♦ Generally, objective tests can be scored on the basis of the key. This involves less time. Moreover, the pupil can record his response definitely and briefly in minimum of time.

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♦ Since the pupil is to give correct response which is correct for all times, the pupil cannot bluff the examiner as may happen in case of essay-type tests in which sometimes it is not possible for the examiner to read the whole answer.

♦ In essay-type tests, sometimes, even good students cannot do well because their writing speed fails them. Objective-type tests eliminate the fact that one pupil can write more material than another in the same length of time.

Possible Limitations of Objective Type Tests

- Objective tests measure only factual knowledge. They fail to measure intellectual skills, such as ability to interpret, ability to analyse critically and ability to solve problems.
- Objective type test items are often ambiguous, particularly for the better students, and therefore penalize them.
- Objective type tests have a negative effect on teaching, since they encourage the student to learn small bits of knowledge rather than broad understandings and since they discourage writing efforts on the part of the students.
- It needs careful thought and enough time to write objective type tests. All teachers cannot prepare highly objective items.
- Objective tests are quite valuable when they are available for class room use in printed or in mimeographed form. But this would mean considerable cost which most schools cannot afford.
- Objective type tests encourage the student to engage in guessing. Once this habit is deeply engrained, it will be hard to change and this may very well yield serious consequences.

Construction of Achievement Test

A test can be made an effective instrument of evaluating the achievement of objectives, the content and the learning activities. We should not evaluate only the content; we have to 3evaluate the total behaviour of the pupils. For this, the achievement of the objectives of all the three domains (cognitive, affective and psychomotor) has to be evaluated. Tests can be conducted at different times during a course.

They are:

- At the end of teaching a daily lesson;
- At the end of teaching a unit;
- At the end of the term;
- At the end of the year or curriculum.

The time factor for testing depends upon the nature of the objectives to be tested. Knowledge objective can be suitably tested either in the course of teaching or at the end of teaching a lesson. The objective of understanding or comprehension requires a comparatively longer period for its testing. Objectives of application and skill require a sufficiently longer period for their testing. The long ranged objectives, such as interest, attitude and appreciation take a lot of time for their development. These objectives can well be tested at the end of the year or when the curriculum has been completed.

Teachers should exercise their discretion in testing the objectives, keeping in mind the nature of objectives in relation to the time factor. They should change the weightage of the objectives to be tested at different times. Testing, if properly done, leads to good learning. Testing should be pre-planned, systematic and scientific. Planning of lessons will give the teachers a full idea of how much weightage is to be given to the content, the objectives, the form of questions, etc., while planning the tests.

The preparation of a good test is a systematic process having well defined stages. The important steps envisaged in the preparation are as follows:

1. Planning of the test
2. Writing the test items

3. Reviewing and editing
4. Arranging the items
5. Providing directions
6. Preparing the scoring key and marking scheme
7. Administering and scoring the test

1. Planning of the test:

Test planning encompasses all of the varied operations tat go into producing the test; but it must also involve careful attention to item difficulty, to type of items to directions to the examiner.

a. Preparation of design:

Designing is the first and most important step in the construction. It is at this stage that we plan to build in the test important qualities-Validity, Reliability, objectivity and practicability. In order to accomplish that, the test constructor has to take a number of decisions regardingthe selection of objectives, content, form of questions, the difficulty level of test items and the weight ages to be allotted to the objectives to the comment and the forms of questions.

b. Identification of the objectives and allotting weightage to the objectives:

Identification of the instructional objectives and stating them in terms of specific observable behaviors. After the objectives are identified and stated the test maker has to decide the relative weight in the test. The important fundamental principal to be observed here is that the test should reflect the actual emphasis being given to various mental process enduring instruction. There cannot be any cut and dried formula for assigning weights to various objectives.

Table showing the weightage allotted to the objectives:

S.No.	Objectives	Marks	Percentage
1.	KNOWLEDGE	5 8	20
2.	UNDERSTANDIN	10	32
3.	G APPLICATION	2	40
4.	SKILL		8
	TOTAL	25	100

c. Selection of the content and allotting weightage to the content:

It becomes very necessary to decide the weights to be given to different parts of it. As the whole syllabus cannot be covered through any single test a convenient number of units can be selected for testing. When this is done a decision about the weights to be given to those units has to be taken so as to represent the actual emphasis on them in instruction. In assigning relative weights to units a number of factors will have to be taken into account. How important is the unit in the total learning experience? How much time was devoted to it during instruction? Although there are a number of such considerations, the easiest method is to decide the weightages on the time required to teach various units.

Example

Table indicating the weightage given to three units namely polynomials, functions and

quadrilaterals.

S.No	Unit	Marks given	Percentage
1. *7* ***3.***	Polynomials Functions Quadrilaterals	10 8 7	40 32 28
Total		25	100

e. selection of the form of question and giving weightage to the questions:

Decide about the form of questions to be used, the number of questions to be chosen and the relative weightage to be given to each form.

Example

The weightages to different forms of questions could be as follows.

S.No.	Fonn	Murks given	Percentage
1. *> 3.	Long answer(L.A) Short	***b*** 9 10	24 36 40
	Total	25	100

f. distribution of difficulty level:

A decision also has to be taken concerning the distribution of difficulty level. The distribution of difficulty level in a test will depend upon the purpose of the test as also on the group of students for whom it is designed. To get optional discrimination through a test most of its question should be of average difficulty level. A few questions here and there may be easy and difficult.

Example

S.No.	D ; level	Percent;
i z. 3.	difficult questions average	15 70 15
	Total	100

g. **preparation of Blue print:**

Preparation of the Blue print refers to the final stage of the planning of a test. It is a three dimensional chart showing the weightage given to the objective, content and the form of questions in terms of marks. It is also called as a table of specifications as it related outcomes to the content and indicates the relative weight given to each of the various areas. The blue print helps to improve the content validity of teacher made tests. It defines as clearly as possible the scope and emphasis of the test; it relates objectives to the content; it provides greater assurance that the test will measure learning outcomes and course content in a balance manner. The major and final responsibility will be that of the teacher, who the

decision maker is. To be of the utmost benefit, it should be prepared well in advance. It would thus assist the teacher in organizing his teaching material, serve as a monitoring agent, and help keep the teacher form straying off his instructional track.

Example

Objectives	Knowledge			,,.__.. ----- ---			Application			Skill			Total
Content	**L.A**	**S.A**	**0**	**L.A**	**S.A**	**0**	**L.A**	**1[A]-**	**0**	**L.A**	**S.A**	**0**	
Polynomial			**1(2)**		**3(1)**	**KD**		**3(1)**	**1(1)**				**10**
Functions			**1(2)**		**3(1)**	**3(1)**			**1(2)**				**8**
Quadrilateral			**KD**				**4(1)**			**2(1)**			**7**
Total	*W*i*			**8**			**10**			**2**			**25**

-

The number inside the bracket indicates the number of questions and the number outside the bracket indicates marks allotted to each questions.

Scheme of options:(choices)

There are two types of options: They are external options and internal options

In case of external options, there are various methods.

- The students are asked to attempt any six out of the given ten questions. This is over all options.
- The students are asked to attempt any tow from the first section, any four from the second section and any fiver from the third section. This is section wise options.
- The students are asked to answer either one question or the other alternate question. This is question wise options.

In case of internal options, the option is given within a question for example; write an essay on any two of the following.

Options should not be given, because the purpose of a unit test is to have complete evaluation of the achievement of objectives of the content and the learning experiences. This purpose is not served if options are given. It is extremely difficult to set two alternative questions or two alternative groups of questions, which measure the achievement of the same objective, the same content or the same learning experiences. The scheme of options may develop some wrong practices among students. They may omit some content. The scheme of options in a test paper influences even the teachers training.

Sections in the question paper

Depending on the type of question, the question paper can be divided into sections. One section can include essay type questions, another section short answer type and the third section objective types. Separate marks for each section should be mentioned. For each section, separate instruction also has to be given.

2. Writing the test items:

Write the test items according to the table of specifications. He should take up each cell of the blue print and draft an item taking care of the various dimensions the objective, the content and the form as laid down in the blue print.

While teaching a lesson, the teacher can prepare questions also. Continuously this work has to be done. When the teacher asks questions in the class room or when the teacher conducts a test, the students respond to them. From the responses of the students multiple choice items can be coined.

Depending on the concept, the teacher teaches in the class, different types of questions can be prepared in the sub concepts. The same item can be presented in different forms and kept ready to be used at any time. So question banks should be prepared by the teachers of the same subjects. Newly prepared questions can be added to the existing questions in the question bank every day, every month and every year.

If an item bank is ready and if a blue print is ready, the only job that remains to be done by the teacher is to select test items form the item bank in accordance with the design of the blue print. A lot of effort is needed to coin better test items. It calls for a mastery of the content and practice in wording test item. But once they are ready, editing a question paper will become easier.

3. **Grouping of test items**

While grouping test items, the following points are to be considered.

- All the obj ective type items should be grouped in one sections, while the short answer in the next section and essay type items should be grouped in another sections.
- In the section of objective type items, the items having the same format, examples yes-no type, true-false type, matching type, multiple choice type should be grouped together.
- Items in each section should be arranged in order of their difficulty, as far as possible.

Sections in a question paper

Generally, objective type items are grouped under section A and short answer type questions under section B and essay type questions under section C.

4. Reviewing and editing:

The pool of items for a particular test after being set aside for a time can be reviewed by the help of experts. The following are to be considered:

1. Does each item measure an important learning outcome included in the table of specification?
2. Is each items appropriate for the particular learning outcome to be measured?
3. Does each item present a clearly formulated test?
4. Is the item stated in simple, clear language?
5. Is the difficulty of the item appropriate for the students to be tested?

6. Does each item fit into one of the cells of the blue print?

5. Arranging the items:

The items should be arranged so that all items of the same type are grouped together. The items should be arranged in the order of increasing difficulty. It may be desirable to group together items which measure to same learning outcome or the same subject matter content.

6. Providing directions:

The direction should be simple and concise and yet contain information concerning each of the following:

Purpose of the test, time allotted to complete the test, how to record the answers, whether to guess when in doubt about the answer and marks allotted for each question. General instructions may be given at the beginning of a question paper.

e.g: This paper consists of A, B and C sections.

- ♦ Answer any two from section A: answer any four form section B and all questions are compulsory in section C.
- ♦♦♦ Specific instruction related to each section may be give at the beginning of each section.

e.g: Answer toe section A should be given on the question paper itself.

e.g: Answer to each question under section BN should not exceed one page.

7. Preparing the scoring key and marking scheme:

It becomes necessary to prepare some other important accessories in the form of scoring key for objective type questions and marking scheme for supply type questions.

Scoring key refers to the prepared list of answers to a given set of objective type questions. The examiner simply compares the answers given by the students with these in the scoring key and thus arrives at the marks to be awarded to the students.

A marking scheme is essential in the case of short answer and essay type items: the following are the important items in a scheme.

- ♦♦♦ Points or steps expected in the answer.
- ♦ Description of each point or step expected in the answer.
- ♦ The weightage to each of these points or steps.

Advantages

- ♦ A marking scheme helps the examiner to bring about a uniformity of standard in assessing and there by increases the objectivity of the est.
- ♦ Many examiners may be judiciously involved in assessing answer books. It will be of help to them.

A scoring key is essential in the case of objective test items. In the scoring key, the item wise correct response in terms of tits number is to be mentioned.

Marking scheme in mathematics will be desirable not only to analyse the solution into important stages and to distribute marks over them but each stage my be looked upon from the point of view of the

method involved as also from expected accuracy. The marks for each stage therefore may be divided into two components; marks for the method and those for accuracy.

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REVIEWING THE QUESTION PAPER

The question paper is to be reviewed because this will enable the paper setter to remove any flaws in the question paper.

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a. Question wise analysis
b. Critical evaluation of the test
c. Item analysis

a. Question wise analysis

This is done in order to know the strength and weakness of a question paper more thoroughly; totally the question paper with the blue print; to determine the content validity of a test and to give satisfaction to the paper setter that the paper exactly agrees with the design.

Each question is analysed into objective, specification, content type or form of questions, estimated difficulty level, approximate time required and marks allotted. These details are generally collected in a tabular form.

b. Critical evaluation of the test

Before printing, the question paper should be subjected to a critical revision by subject experts who may be able to discover its shortcomings. The following points should be considered while evaluation of the test.

- ♦ The directions to students should be as clear, unambiguous, complete and concise as possible. Nothing should be left for guess work.
- ♦ There should be an adequate coverage of all the topics, specifications and learning activities.
- ♦ There should not be any items that test only trivial points.
- ♦ All the criteria that are necessary, while preparing the obj ective type test items, should be fulfilled.
- ♦ Similar types of items should be placed together to speed up scoring ad evaluation and enable students to take full advantage of the mind set imposed by a particular item format.
- ♦ The test items should be graded, that is, arranged in order of their estimated difficulty level.
- ♦ Classroom tests are power tests. The time allotted should be such that at least 95 percent of the students in the class should be able to completer the test within the stipulated period of time.
- ♦ A marking scheme and scoring key should be kept ready.

a. Item analysis

After an achievement test has been constructed and administered in the classroom, the scores obtained by the students maybe taken on their face value and recorded to suggest which students are good, which average and which stand low in merit on the combined list. One may leave the results as such, without bothering about the suitability and stability of the various test items included in the classroom testing programme of pupils achievement. But, on the contrary, it is bound to pay rich dividends if the relevance of each test item is obtained, that is, if the relative difficult and discriminating power of the test

item is ascertained.

The process of determining the relative difficulty and discriminating power of the test item is known as item analysis.

Item analysis is an analysis of responses made to teacher made tests by pupils in the class, an inspection of the percentage of students who correctly answered each item, the process which reveals areas in which instruction was especially good or especially poor. The process of item analysis is best applied to objective type test items; it can also be applied to essay type tests, provided they are scored by the analytical method, which is a difficult process.

Functions of Item Analysis

- Item analysis provides valuable information concerning student performance.
- Item analysis provides the effectiveness of teachers.
- It also describes about the test-item characteristics.
- The primary purpose of the item analysis is to improve the quality of the test item for further administration.
- The second purpose of considerable importance is to study the strengths and weaknesses in the academic achievement of the students who have responded to the test item.
- Another advantage of the items analysis is that by studying the nature of the incorrect responses, the teacher is able to gain a superior view of the relative position of each student with regard to the material being tested.
- The immediate purposes of an item analysis are to determine the difficulty and discrimination of each item.
- When an item analysis is performed on a test, one is almost certain to gain additional important insight into the examinees thinking, understanding and test taking behaviour.

The process of item analysis should improve an instructors skills in test construction beyond that possible otherwise.

Item Difficulty and Discrimination Power

The two important characteristics desired for a test item are its difficulty level and discriminating power.

Most test experts believe that satisfactory evidence concerning these characterists can be obtained by considering the performance of only a portion of the group, namely, those who performed very well on the total test(the high group) and those who performed very poorly(the low group). For statistical computation, the high group comprises the upper 27% of the total group and the low group, the lower 27%. The computations of difficulty and discrimination indices are based on these two fractions of the total group.

Item Difficulty

The difficult level of a test item means how well the test item distinguishes between students who know more and the students who know less.

In the words of Prof. Ahmann, "By test item difficulty is meant the percent of students who correctly answer to a given test item."

The information regarding the test item difficulty can be of great help to the teachers if his test is to "yield scores on the basis of which students are to be ranked in terms of their achievement". Such information also provides some indication of "the extent to which the item is doing its job."

The following formula is used to determine the difficulty level of a test item: V »,

where p is the item difficulty index

Nr is the total number of students in the combined high and low group who give correct answer to the test item.

Nt is the total number of students in the combined high and low groups.

The difficulty level index may range form 0 to 1.

Administering and scoring the test

This is the last step. The main thing is to make certain that all students know exactly what do and then to provide them with the most favorable conditions to do it. After the administration of the test, the scoring can be done with the help of the scoring key and marking scheme.

Standardized Achievement Tests

A standardized test is a test which comprises of carefully selected items, having been given to a number of samples or groups under standard conditions and for which norms have been established after careful evaluation. It is a specially designed test keeping in view the instructional objectives. It is the product of the joint efforts of a number of persons, including test experts. It is produced by some test publishing agency. The standardized tests have high validity, reliability and discriminating power.

Merits of **Standardised Tests**

- The standardized tests are accompanied by carefully established norms. Norms provide basis for comparing the achievement of pupils in the same age and grade. Pupil's achievement may also be compared with national norms to get a more realistic picture of the achievement of pupils.
- Standardized tests are constructed after carefully defining instructional objectives. They possess greater objectivity.
- Test experts devote considerable time in their planning, try out and evaluation. Thus these tests are superior in all technical aspects.
- Since items of standardized tests are well written, these tests possess high degree of validity and reliability.

Comparison between standardized and Teacher made Achievement Test

Characteristics	Teacher Made Achievement Tests	Standardized Achievement Test
Sampling of content	Sampling and content are both determined by classroom teacher.	Content is determined by experts after extensive investigations of existing syllabi, text books and programmes; sampling of content is one systematically
Construction	Constructed in a hurried and haphazard manner; often there is no blue prints, no item tryouts or item analysis and revision.	Constructed after carefully defined instructional objectives; involves blue print, item tryouts, item analysis and revision.
rrms	Only local classroom norms are available.	Besides local, national, school, district norms are available.
^ministration and 'ring	No uniform directions in this regard are available.	Specific instructions standardize administration and scoring procedures.
poses and Use	Best suited for measuring particular objectives set by teacher and for intra class comparison.	Best suited for measuring broader curriculum objectives and for inter class, inter school and national level comparison.

Diagnostic Test

The Meaning and Purpose of Diagnostic Testing

Literally diagnostic testing stands for the testing and evaluation programme carried out for the diagnosis of something. In case one has some problem regarding his physical or mental health he is subj ected to one or the other land of testing, (blood test, urine test, stress test and blood pressure, ECG, X-ray checking etc) for diagnosing the nature as well as roots or causes of his ailments. The results of all such diagnoses then form part of the remedial or treatments programmes for helping the individual from getting rid of that physical and mental health problem. Similar is the case with the diagnostic testing and evaluation programmes carried out in the field of education. Educational efforts are aimed to bring desirable behavioural changes for an all round development in the personality of an educand.

Most standardized and teacher made achievement test are designed to give and indication of how far the student has progressed towards the accomplishment of specific objectives measured by the test. These objectives, however are grouped into broad categories. They cover a broad area and result

in a total score which reflects over all achievement in the area tested. Thus the teachers can say that a pupil is doing well in arithmetic or poorly in arithmetic, but they do not know why nor do they know what are the concepts causing difficulty. It will identify students who are having relative difficulty in an area but it will not identify the causes of the difficulty.

Such achievement tests serve a useful function, but inorder to help the student with a disability, the teacher will need to analyse the nature of the difficulty and the causes for the trouble. There are tests which have been devised of to provide information about the specific nature of pupils difficulties in given subject areas. These tests are called diagnostic tests.

Thus diagnostic test measure some what narrower aspects of achievement than do survey tests. Diagnostic test yield measures of highly related abilities underlying achievement in a subject. They are designed to identify particular strength and weakness on the part of the individual child and within reasonable limits to reveal the underlying causes.

The diagnostic test attempt to break a complex still like computation into related parts such as addition, substraction, multiplication and division and to provide separate measures of these subskills. Such measures can help the teacher to locate the sources of difficulty form which constructive action can be taken. Hence we are going to diagnoise and find remedies for learning mathematics.

However many times these efforts may not yield the satisfactory results or the individual students may not be duly benefited through such efforts resulting into one or the

other kinds of behavioural problems or educationally failure. We are alarmed when we find student turning into a problem child or observe him lagging behind in his studies related to one or more subjects of the school curriculum. Here comes the need of diagnosing his behaviour and state of educational progress in one or the other subjects of the school curriculum.

Going in this way, the need of diagnostic testing arises in the subject mathematics specifically at the time when a particular student exhibits some or the other signs and symptoms of his failure or difficulties with regard to the learning of the subject mathematics. Why is one subjected to repeated failure in the subject mathematics? Why is he feeling difficulty in learning a particular concept or skill in one or the other branches and topics of mathematics? Why is he not attending the classes in the subject mathematics? Why does he create fuss or problems in the mathematics class? Why does, he hate the teacher of mathematics and his subject? The list may be quite exhaustive with regard to such day to day or occasional problems faced by a teacher of mathematics with his one or more students.

Surely like a physician or psychiatrist, here he has to resort to the methods of diagnosing the learning and behavioural difficulties of his students for chalking out some remedial programmes aiming to help them in getting rid of their difficulties and problems.

Looking in this way diagnosting testing in mathematics may be defined as a testing or evaluation programme carried out by a mathematics teacher for diagnosing the nature and extent of the learning difficulties and behavioural problems of an individual or group of students along with the inherent causes by chalking out suitable remedial programmes aimed to help them in getting rid of their difficulties and problems.

What is Diagnosis?

Diagnosis has been borrowed from the medical profession where it implies identification of disease by means of patients symptoms. The word diagnosis is used more or less in the same sense in education. The only difference perhaps is that in medical diagnosis it is physical or an organic break down that is examined, while in educational diagnosis it is the failure of the process of education or

learning that is located and attended to be remedied. We may say that educational diagnosis is the determination of the nature of learning difficulties and deficiencies. Of course it cannot stop only at the identification of weaknesses in learning but has to go a litter deeper to locate their causes and also suggest remedies for getting rid of them.

Characteristics of Diagnosis

- Diagnosis must be made in connection with worthy objectives of a good educational programme.

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- It should be obj ective, reliable and valid.
- It should be as specific as the desired outcomes permit and as the possibility of localization of symptoms allows with in the limitations of practicability.
- It should yield results that would be comparable over a period of time and between groups of students.
- It should be sufficiently precise to note progress during small units of time.
- It should be comprehensible.
- It should be appropriate to the educational programme.
- The person making the diagnosis must understand the educational programme and be familiar with the fundamental problems of children.

Need for Diagnosis in Teaching

1. To identify and gain information about the learning difficulties of individual students in a specific concept or content.
2. To provide a specific remediation or remedial instructional measures to help the students overcome or correct the learning difficulties.
3. To individualize instruction and enhance mastery learning.

Purposes of Diagnosis

The following are the purpose served by diagnostic tests:

- To point out inadequacies in specific skills.
- To locate areas in which individual instruction is required.
- To furnish continuous information inorder that learning activities may be most productive of desirable outcomes.
- To serve as a basis for improving instructional methods, instructional materials and learning procedures.

LEVELS OF DIAGNOSIS

Classification:

It is the process of sorting out students into groups particularly of under achievers and low achievers in the context of educational diagnosis. The reference point for each student should rightly be his own expected achievement and we have to sort out the students with regard to their levels of expected achievement. If they have not reached their levels they will be the ones who are in need of remediation. However, if have crossed their level, we may plan some enrichment programme help them improve their achievement further.

Finding the nature of difficulties:

In this level of diagnosis we have to pinpoint the specific areas where they experience of difficulties. Achievement test, unit test etc, can be used for this purpose as they cover as many learning point as possible. Do the item analysis. The analysis helps to locating the weaknesses of the students as revealed by the test. If most of the students do poorly on a particular learning point, we get an indication that something is wrong with the instruction relating to the point. But this type of analysis gives no ideas regarding the causation of these errors which will have to be sought by the other means.

Aetiology:

This is the most difficult stage in diagnosis. The main difficulty of this stage lies in the fact that test appraise only the products of learning and not the process of learning. They may establish where the break down in learning has taken place but can seldom tell anything about the causes of it. These causes are generally varied and complex in nature. We have to seek for them in different areas some of which need not be connected in anyway with classroom, instruction or school. Broadly speaking under achievement may be due to factors within the student or environment factors outside the control of the students or a combination of the two. Most of the causes internal to the student may however be located in the areas of scholastic aptitude, retardation of basic skills, study habits, physical factors and emotional factors. Interviews, check lists, rating scales, questionnaire etc can be used to sort out the areas.

Remediation:

When the basis of pupils difficulty is understood, we come to the stage of applying remedial measures. There is however, no set pattern and no cut and dried formulate for remediation. In some cases it may be a simple matter of review and reteaching. In other, and extensive effort of improving motivation, correct emotional difficulties and over come deficiencies in work study skills maybe required. The hared fact is that there are no patent remedies in educational practice for two students having the same learning difficulty may

have suffered it because of different causes and may have to be tackled differently. Moreover since each subject has the own genius and personality, remedial programmme have to be planned accordingly. Despite the different methods and techniques needed in remediation, there are certain guiding principles that apply to all subject areas and provide a frame work in which the teacher can operate. Remediation should be accompanied by strong motivational programme. Remediation should be individualized in term of the psychology of learning. There should be continuous evaluation giving the pupil a knowledge of results. Remediation programme may not always need a separate time allocation for them. But they will always mean some extra work for both the teachers and the affected students.

Prevention:

Prevention is better than cure in education as else where. A programme of diagnostic testing should help an imaginative teacher in getting an insight into the types of possible errors that are likely to occur in learning their possible causes and the ways of preventing them in future. Thus educational diagnosis does not and should not end at remedial measures, but also should become a means for improving instruction modifying its curriculum and also for retaining instructional materials. The real importance is rather in the prevention of its reappearance elsewhere under similar conditions. Any weakness identified should afford the basis for decisions calculated to reduce the probability of their recurrence in the future.

Remedial Teaching

Remedial teaching employs a greater variety of instructional and learning correctives to overcome the constraints experienced by the students during learning a particular concept or topic. It assumes that quality of instruction could best be defined in terms of:

i. The clarity and appropriateness of the instructional cues for each student

ii. The amount of active participation in and practice of learning allowed to each student and

iii. The amount and variety of reinforcements and feedbacks available to each student to complete his unit oflearning.

The teachers may use various instruction/teaching correctiv4es. Viz., small group problem session, individual tutoring, alternative learning materials including textbooks, workbooks, audio visual techniques, academic games and puzzles and re-teaching.

Small group problem sessions involving three to four students with very different learning difficulties work best with elementary school children. While teaching these children we may observe that they lack both independence and perseverance, which are required to complete a learning task by themselves. As a remedial measure, small group problem sessions involved three to four students provide a specific block of time when students are formerly constrained to attempt to complete their learning tasks.

Individual tutoring is also an effective learning corrective remedial measure, which can be used for correcting individual learning problems of students. It can be done either by the teacher himself or by someone other than the teacher. It is advisable that the tutoring should be done by someone other than the teacher who can bring new perspectives to students in the teaching learning process.

Alternative learning materials and textbooks already available or easily acquired by the school are also useful remedial materials. You may adopt a textbook or some instructional/ learning material for course use, which is not of consistently high quality in explaining particular points and processes. You can use alternative learning material/textbooks to fill in these gaps and provide better explanation of difficult learning spots.

Workbooks are most useful remedial tools especially for students who have great difficulty in grouping ideas and processes from a highly verbal type of instruction. Workbooks provide drill and the specific problem solving practice, the students need for learning.

Audio visual materials, viz., film strips, motion pictures, classroom demonstrations, charts and illustrations can be used in the case of students who have difficulty in grasping material presented in a verbal abstract instructional mode.

By diagnosing we may identify the learning deficiencies and hard spots. After this diagnosis, it would be advisable to re teach the entire class.

All these remedial correctives are added to supplement and not to replace the original instruction and instructional materials used in the class. They may be viewed as helping measures to be used for a student at those particular hard points where his original instruction was not of optimal quality.

The main focus of diagnostic testing and remedial teaching is to build into the instruction the feedback devices so as to point out deficiencies in student learning of a given instructional unit and to suggest the alternative instructional correctives required to overcome them. Re-teaching of selected aspects of an instructional unit, small group co-operative study sessions, individual tutoring, and use o various alternative instructional materials and methods may prove effective and useful instructional correctives by teachers for the students. These correctives attempt to provide each student with instructional cues, the

learning participation-practice, and the reinforcements, which are best, suited to his characteristics and needs. With the help of diagnostic testing followed by effective remedial correction procedures. We can transform classroom instruction of any initial quality into instruction of optimal quality for each student.

Distinguishing Diagnostic Tests from the Acheivement Tests

A diagnostic test is primarily carried out for knowing about the nature and extent of the learning difficulties and weaknesses of a student or group of students in a particular learning area, subject, topic or concept. Here all efforts are concentrated on the search for the areas of weaknesses (or strength in the case of above average) and error analysis (for knowing the probable causes of weaknesses) resulting into one or the other form of remedial teaching to help the needy students in getting rid of their weaknesses and learning difficulties.

The search can be better made if the area and scope of this search is kept small and extensive as well as vigorous efforts are made for such search. Similar is the case with diagnosis in Mathematics. Here the task of diagnosis should be broken into simpler and smaller curriculum units and students should be asked a sufficient number of questions, preferably short answer on very short answer type including completion type objective questions for going into the depth of their areas of difficulty and weaknesses.

With the help of what has been above said about the diagnostic and achievement tests, we can be able to distinguish them from each other in the manner given below:

1. The achievement tests while focusing on knowing about the level of the learner's achievement may only provide a hint or clue about the learning difficulty or weakness of a student, the diagnostic testing goes, deeper in concluding about the nature, extent as well as reason for such learning difficulty and weakness.
2. Sampling of questions in an achievement test need detail and not to be too exhaustive to cover each and every minute points with regards to the coverage of the contents. In diagnosis on the other hand, we have to go quite in depth. Therefore, here the coverage of the subject matter is more detailed and exhaustive, though based on a smaller area than an achievement test. Diagnostic tests, have therefore, to be much longer than the achievement test to make necessary subtests sufficiently reliable.
3. Although the results of an achievement lest may provide a signal or warning about the poor performance of a student yet these are unable to provide necessary base for the planning of a remedial programme. Diagnostic testing on the other hand necessarily leads to the formulation of a well thought remedial programme aiming to help a particular learner from getting rid of his learning difficulties and weaknesses.
4. The performance of the individual student in a diagnostic test is subjected to a careful analysis with regard to the common as well as specific error made by him for concluding about the exact nature, extent and causes of his learning difficulties and weaknesses in a particular learning area. The results of achievement tests are not subjected to such error analysis as these are primarily utilised for measuring the levels of the achievement or performance of the students with regard to their gain in learning during a specified period.
5. In diagnostic testing we require more detailed and exhaustive content analysis i.e. breaking of the unit into multiple learning points and then arranging these points in a hierachical order without breaking their continuity. We can't break this continuity in case of the diagnostic test as the test in itself will become faulty by becoming unable to untouch the weaknesses of those students who are weak and

deficient in respect of one or the other learning points omitted in the test. Achievement test on the other band, do not require such type of compulsion for arranging the testing learning experiences into some hierarchical order and emphasize upon each and every learning points by maintaining their continuity in a hierarchical order.

A. **Planning.**

An appropriate thorough planning is very much essential for the construction of a diagnostic test. Usually it may involve the following considerations:

(i) Identifying the areas of weakness or learning difficulties

First of all the need of constructing a diagnostic test should be properly identified. It may be based on the findings of the achievement test, classroom drill and practice Work, homework and assignments, classroom behaviour of the students etc. The performance and behaviour of the students of the class (or a particular student) during such evaluation encounters may provide a clue or evidence of some or the other types of weaknesses and learning difficulties 'suffered by the students (or a particular student) in one or the other learning areas of mathematics.

(ii) Isolating a unit, sub-unit or concept for diagnosing in depth

Suppose, section B of the class IX (or one or the other particular students) has demonstrated a quite poor performance in the subject mathematics. What are the different weak areas.

Certainly, it needs a careful analysis of the results of performance tests and academic encounter with the students: Let us further assume that this analysis has led us to conclude that particular student or class is lagging behind or feeling difficulty in one or some particular units of the IX class mathematics curriculum. Now these units may work as a base for the construction of an achievement test. However since diagnosis is to be carried out in quite detail and depth, therefore it is not advisable to take all these units or even a full unit for the construction of diagnostic test. Let us therefore try to take only one unit, or a sub-unit and single concept for the construction of a diagnostic test. In this way several sub-unit or single concept diagnostic tests may be constructed and then joined and combined into composite diagnostic test aiming to locate the learning difficulties and weaknesses of the students in the whole unit or units of the total prescribed curriculum in mathematics.

(iii) Content Analysis

The Contents of the sub unit or a single concept should then be further analysed to determine:

- the pre-requisite behaviour *i.e.* the previous knowledge, skills etc. needed for the learning of a particular sub-unit or a single concept.
- the expected behaviour demonstrated by the learner after learning the contents related to that unit or a single concept.

(iv) Deciding about the nature of the items of the test

A proper decision should then be taken about the nature and numbers of the items of the diagnostic test since it should be as exhaustive, detailed and lengthy as possible, hence there must be more weightage to the short and very short answer type questions in comparison to the large and essay type questions. It is preferable to use completion type items in the test. However, for diagnosing the actual practical difficulties, of the students in solving the mathematical problems, a sufficient number of problems related to the know how of the knowledge, skills and application of the mathematical concepts should necessarily be included in diagnostic test of mathematics.

(v) **Taking decision about necessary administrative measures**

It is better to take decision regarding various administrative functioning of the test before sitting for its construction like, the time limit, direction for the proper administration of the test.

B. Construction of the Diagnostic test

In view of the things planned at the planning stage now attempts should be made to select appropriate items for being included in the proposed diagnostic test. This selection is mainly focused on the following three aspects.

1. The nature of the contents of the sub-unit or single concept.
2. The pre-requisites behaviour in terms of the previous knowledge, skills needed for learning the sub-unit or concept.
3. The expected behaviour i.e. knowledge, skills and application etc. acquired by the students after studying the contents of the sub-unit or single concept.

For the illustration purpose let us choose one of the sub-units of major unit mensuration of the subject mathematics namely "area of a circular figure" for the construction of a sample Diagnostic test.

Diagnostic Testing and Remedial Teaching in Mathematics

It can be easily concluded from the above discussion that diagnostic testing and remedial teaching are inter-related and complementary to each other. Each is based on and results into the other. One resort to diagnostic testing for searching the appropriate remedial instructions. However, the diagnosis not followed by the remedial treatment is useless. Similarly remedial task not based on the diagnosis of the nature and extent of the weaknesses may prove not only the wastage of the resources but can also prove dangerous to the well being of the affected persons. It is therefore, essential that diagnostic testing in mathematics should necessarily be followed by the suitable remedial 'teaching. In fact neither diagnostic testing nor remedial teaching should ever be considered in isolation. They should form a part of a cycle known as Diagnostic testing and Remedial teaching cycle which may be considered to involve the following processes for its complete execution.

1. Diagnostic testing for knowing the child's weaknesses and learning difficulties in mathematics.
2. Hypothesizing the probable causes for these weaknesses and difficulties.
3. Applying remedial teaching for removing these weaknesses and difficulties.
4. Evaluating the outcomes of the remedial teaching
5. Continuing to repeat the above four processes to achieve desired success in removing the diagnosed difficulties and weaknesses

A close analysis of the above mentioned Diagnostic testing and Remedial teaching cycle may at once reveal that here the cycle starts With the application of its first step, Diagnostic testing—testing undertaken for diagnosing the weaknesses or learning difficulties of an affected student (who has been identified so as a result of achievement tests teachers observations, etc.) Remember here, the diagnostic testing or need of administrating diagnostic tests and other diagnosis measures arises only when something wrong and serious is reported about the progress of the child in the learning of one or the other subjects By applying the diagnostic testing the efforts are made to know the nature and extent of ones weaknesses and difficulties in the learning of that particular subject say mathematics

Once the weaknesses and difficulties regarding the learning of a particular concept, knowledge and skill area etc are identified, efforts are then made to list out the probable causes responsible for

these weaknesses and difficulties.

The listing of such probable causes then may be made a base for building a remediation program. The affects or outcomes of such remedial program or remedial teaching are then evaluated to see whether or not we have got desired success in getting rid of the child

from his identified weaknesses or difficulties in the learning of mathematics. In case we don't get the desired success indicating the ineffectiveness of the remedial teaching, then the diagnostic-remedial cycle must be re-initiated and the processes of diagnosis, hypothesizing remediation evaluation etc must be brought into operation again for the attainment of desired success in helping the child to get rid of his learning difficulties or weaknesses in one or the other areas of mathematics, With all what has been discussed so far, one can assume that diagnostic testing and the subsequent remedial teaching is exclusively meant for he students who suffer from weaknesses or learning difficulties in one or the areas of learning a subject or those who are under achievers, slow learners backward, learning disabled or suffer from the learning or behavioural problems with regard to the teaching-learning of a subj ect.

However it is not suffice as the term diagnostic testing and remedial teaching can be equally applied in the case of students who are gifted, creative and meritorious. Diagnosis helps in diagnosing the weaknesses as well as strengths, therefore diagnostic testing may properly carried out for knowing what is appreciable, original, creative and above average among the geneous creative and meritorious students.

Based on such diagnosis, the efforts should be made to harmness on their potentialities and maximise their strength for helping them further to grow and develope in the better way.

Therefore, it is no harm in carrying out the diagnostic testing and remedial teaching programmes for the gifted, creative or meritorious. Rather it will be a boon for them in one or the other ways. However, as you know necessity is the mother of invention. We try only when we feel the urgency of trying, similarly here, we resort to diagnostic testing and remedials teaching only when the necessity of doing such work is urgently felt. Since this urgency in mostly felt in the case of those who are weak or feel difficulty in learning, the diagnostic testing and remedial teaching programmes in mathematics are then usually carried out only with the students who are identified with one or the other types of learning difficulties and weaknesses in the study of the subject mathematics. In the pages to follow thus we will be concentrating our efforts for evolving diagnostic testing and remedial teaching strategy only for such children.

REMEDIAL TEACHING IN MATHEMATICS Meaning and Purpose

The term remedial teaching as the name suggests stands for the teaching or instructional work carried out to provide remedial measures for helping the students (or individual student,) in getting rid of their common or specific weaknesses or learning difficulties diagnosed through diagnostic testing or some other measures carried out for such diagnosis.

Diagnosis thus provides a solid base for hypothesizing the general and specific causes underlying the weaknesses or learning difficulties of the students of a class group (or a particular students). It is thus true that as the diagnosis so is the remedy for the removal of the difficulty Let us think over the possible remedial measures in the light of this very assumption.

1. In case the class as a whole is demonstrating a particular type of weakness or learning difficulty then the treatment should be followed by a common remedial programme.

For this purpose, special classes may be organised or special methods and techniques may be employed for making the class understand properly the concepts or skills etc. related to the area of weakness.

2. In case the weakness or learning difficulties diagnosed are of specific nature applicable to the individual students. Then the treatment should also be individual specific. Mathematics is a quite sequential subject. Here the study of a topic is dependent upon the previously learnt topics. In case the student is lagging behind in the learning of the pre requisites essential for the study of the topic in hand, then the treatment in terms of the remedial teaching must necessarily be given for making up the previous deficiencies in learning. The child should be helped individually for this prupose by making use of the methods, techniques and teaching-learning situations best suited to the individuality as well as nature of the content.

In case the learning difficulty felt at present is not the product of the deficiencies in terms of prerequisites or previous learning, then its roots must necessarily lie in the present set up of the teaching learning. The results of the diagnostic testing and adequate analysis of the errors committed by the individual student then may bring on the surface some or the other probable causes for the diagnosed weakness and learning difficulty of the individual students. In the light of such diagnosis a proper remedial teaching programme should be taken into hand.

Any remedial teaching programme includes three major steps:

- Diagnose a learning difficulty.
- Remedy the learning difficulty with a prescription,
- Encourage and support the learner all the while.

Remedial Teaching Programmes.

Let us now initiate he task of such individualized remedial teaching programme based on the diagonised difficulties and error analysis of the responses of the students.

(i)Let us take the case of an elementary class student. He commits the errors in multiplication of numbers in the way as.4 x l = 5 o r 5 x l = 6 , etc. Error analysis may here reveal that he is lacking in terms of the understanding of the basic concept of multiplicative identity n x 1= n. Now the remedial teaching programme in this case may, be proceeded as below:

(a) Use concrete aid material representing a single obj ect repeated 4 and 5 times or a single bundle of the items containing 4 and 5 objects making the students conclude that 1 x4 or 4 x 1 = 4 and 1 < 5 or 5 x 1 = 5.

(b) Making the students practise as below

4x =4 5x =5 4x1 = 5x1 =

x 1 =4

x 1 = 5

(ii) Let us take the case of another students who has been diagnosed to feel difficulty in multiplying a (b + c) or (a + b) (c + a). From such diagnosis it is natural to conclude about the ignorance of the student for the distributive properly of multiplication over addition [(for any three number a, b and c,

a (b+ c) = ab + ac]. Let us further assume that diagnosis in this case also reveals his ability to identify this property with whole numbers, through his written response as 5x47 =5 x (40 + 7)

= (5 x 40) + (5 x 7) = 200 + 35 = 235

It may now lead us to conclude that, in the present case the student is feeling difficulty either on account of the weakness in the programme of his algebra instruction or due to his own inability to transfer the knowledge and skill acquired for using distributive property with whole numbers to algebraic expression.

Thinking on this line a remedial teaching in this case is to be aimed to help the student to generalize the distributive property of multiplication over addition from whole numbers to algebraic expressions. Accordingly the remedial instructional work may be planned through the following comparison table to observe the step by step analogical connection between operations with whole numbers and algebraic expressions

(i) 5x47		(i) a(b + c)
(ii) 5 x (40 + 7)		(ii) a x (b + c)
(iii) (5 x 40) + (5 x 7)	7)	(iii) (a x b)+(a x c)
(iv) (200) + (35)		(iv) ab + ac
(v) 235		(v) sum of ab and ac.

The knowledge of the general applicability of the distributive property of multiplication over addition, then may be further strengthened by helping the students to leam about (a + b)(c + d)=ac + ad+ be + bd, through the step by step comparison of the process with the multiplication of whole numbers like 35 x 47 [expression (30 + 5) and(40 +7)].

(iii) While responding to the item 2 of the part p of the diagnostic test on "The area of a circular figure." let us assume that a student has committed the error in the recollection of the formula. His answer was 27tr instead of 7ir2. Subsequent error analysis, then may give rise to the following hypothesis for his weakness or underlying difficulty in providing correct answer:

(a) The concepts of the circumference and area of a circle are not clear to him.

(b) He has not been able to recollect the correct formula as the same was learned through cramming i.e. memory level teaching-learning instead of understanding level teaching learning.

In view of the above probabilities, the desired remedial instructional work in this case may be proceeded as under.

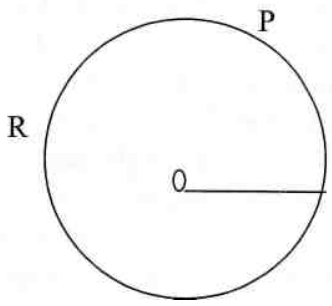
(i) The concept of the length of the circumference and area of a circle will be made clear to the students with the help of concrete examples and laboratory experiments. He will be presented with a circular disk or any solid figure of circular shape. A circle with a

given radius may also be drawn on a white paper. The student may be asked to put a dot or mark on the circular edge of this figure and then, taking it a starting point he may be asked to roll up a piece of thread around the circular shaped solid and then finally measure its length being termed as the length of the circumference of the circle.

Here O is the center of the circle drawn with the length OP as its radius starting from P, the length travelled all along the circular path passing through Q, R and S (taking the whole round-starting from and coming back to P) is known as the length of the circumference of the circle computed by the formulae / = *2nr.*

Area of circle denotes the measurement of the total space available in the circular figure. For the clarity of the concept about the area of a circle some concrete illustrations should be provided. How much cloth or sun mica is needed to cover the top of a circular table? How much space in available in a circular plot? How much coloured paper is needed to cover a circular cardboard figure etc.

The total covered space within the circular boundary of the given circular figure is termed as the area of this figure.



(ii) The students should be made to learn about the establishment of the formula area of a circle = 7tr2 through the use of laboratory method. It has already been mentioned somewhere earlier in the text. Here the given circular figure drawn on a piece of paper is gradually converted into a parallelogram or a rectangle (approximately) to let the students calculate its area as length x breadth. The length of this converted figure measures as equivalent to the half of the circumference of the circular figure i.e. nr and breadth is measured as equivalent to the radius of the circle 'r' giving the area of the converted rectangular figure (or the circular figure in original) as jrr x r = 7ir2. In this way the students are made to learn all about the establishment of the area of a circle, through their own doing and understanding. Such understanding level instruction, then may help the student to get the formulae remembered well or recollect soon (if forgotten) by going through the process of its establishment.

We have tried to mention about some of the remedial measures meant for removing the weaknesses and learning difficulties of the children caused mainly on account of the lapses in the instructional process or learning inability or deficiencies of the students. The roots of the behaviour problems needs to be rectified thus here lies in the cognitive domain of the students. However, there may be so many cases where for the felt learning difficulties or demonstrated weaknesses the physical as well as emotional reasons maybe found more or even sold)/ responsible for a particular child or group of children. The situation like this may be pointed out as under.

1. The child may feel difficulty in learning and progressing well in the subject mathematics on account of his poor health, ailments or physical and mental handicapness.
2. A particular child or the group as a whole may suffer academically on account of the poor physical and learning environment as well as facilities available to him/them is the school as well as in their homes.
3. The child may feel difficulty in the path of his learning and may demonstrate serious backwardness, deficiencies and weaknesses in terms of the learning of the subject mathematics 'purely on account of the emotional reasons. A simple rebuff from the teacher may cause a child to become completely disabled, intellectually and emotionally. The vicious circle of misunderstanding once started between the teachers and taught may develop into a bigger problem of first hating the teacher and than the subject eventually terming a so capable child into a problem or backward child in mathematics. Sometimes such behavioural problems are the product of the over expectations, mishandling and over anxiousness shown by the parents about their children.

There are some cases where the parents were demanding high achievement in mathematics from their child on account of getting him admitted in a very competitive course. They were loosing their patience and getting extremely tense when he did not meet their standards. Eventually to save his face, the child developed an escaping device in the shape of becoming physically ill at the time of testing and examination.

Now in such a case the remedial programme in terms of tight mathematical instructions making the child to work more and more can cause more emotional block to his learning of mathematics. Therefore, here the treatment needs psychological help more than the academic caring. The same happened to this child. The parents were told to behave in a normal way not to be too worried about his mathematics study. The demands for high achievement in mathematics were lessened.

The efforts were made in the school and home to love and accept him with no strings (such as accepting or loving him only when he achieved high in mathematics) attached. Test and examination phobia was also removed to bring positive and favourable changes in the environment. He was allowed to team up with his close friend in test situation. In this way with in due course of time, applying behavioural therapy, the child became normal and began to develop interest in the more serious learning of the subject mathematics.

In this way the corrective measures for the removal of the learning difficulties and weaknesses of the students in the subject mathematics must involve all the aspects and domains of the children behaviour-cognitive, conative and affective (assuming that cause of their problem and deficiency may be cognitive, physical and emotional in nature). Many times, the problems, deficiencies and weakness do not arise on account of a single factor.

There may be a multiple number of causes, academic, physical or emotional in character responsible for one or the other learning difficulties and weaknesses of the students in a particular area or learning aspects of the subject Mathematics. Therefore, a global strategy involving all the possible corrective measures in terms of improved instructional strategy, modifying teaching, learning environment, adjusting the teaching learning according to their physical and mental abilities as well as emotional requirements and seeking guidance from the clinical experts should be adopted for helping the needy students in the task of being get rid of the diagnosed learning difficulties and weaknesses in the subject mathematics.

While planning diagnostic evaluation, the teacher should bear in mind the following points:

- The diagnostic tests should be prepared in accordance with the specific tasks regarding a particular subject or area of a subject.
- There should be large number of items or questions covering various aspects of the relevant subject matter for which the test is designed.
- The questions have to be easily administrable.
- Scoring of the items should be easier and quicker.
- Variety of questions like very short answer type and multiple choice type should be included.
- Different kinds of tests may be used to suit the students with different abilities.

While administering the tests, the teachers should keep in mind the following points:

- The teachers should make their students understand that the tests are to help them to ^improve their learning.
- The tests should be administered in a relaxed environment.
- Students should not consult each other while taking the test.
- The teacher may ensure that the students attempt all the questions.
- Time schedule should not be enforced strictly.

Uses of Diagnostic Tests

- Diagnostic tests serve as guides to the attainments of students.
- They serve as guides to locate difficulties of students i.e., isolating difficulties of students individually.
- They help in dividing students into groups for remedial teaching or special coaching

Diagnostic Tests for Reading and Arithmetic

Diagnostic tests are designed to analyse individual's performance and provide information on the causes of difficulty. Such tests are generally administered in reading and arithmetic.

Diagnostic tests in reading range form group tests to intensive clinical programmes for individual case study. IOWA silent reading tests are short, but widely used group test. The common functions covered by both the batteries are:

- Rate of reading
- Vocabulary
- Sentence comprehension
- Paragraph comprehension

Diagnostic Test in Mathematical Skills

The Compass diagnostic tests in arithmetic comprise twenty tests covering different arithmetic operations.

The diagnostic tests for fundamental process in arithmetic prepared by Buswell and John are individual tests. Each problem in this test is to be solved orally by the subject. When the subject is engaged in the process of solving the problem, his method can be observed. Errors as well as undesirable work habits are recorded on a check list.

A **Sample Diagnostic Test in Mathematics**

Name of the unit— Mensuration

Name of the sub-unit Area of a circular figure

Pre-Requisites.

Students are expected to have previous knowledge regarding the following :

1. They are well acquainted with the circular figures in terms of naming and recognizing their parts like centre, circumference, diameter, radius etc.
2. They are familiar with the concept of area.
3. They are familiar with the concept of circumference.
4. They can recall the formula for computing the length of the circumference of a circle.
5. They are familiar with the concept of n and are able to tell its value as 22/ 7
6. They are able to compute the length of the circumference of a circle of a given radius r

Expected Learning Outcomes

1. They will be acquainted with the concept of the area of a circular figure.
2. They will be able to recall the formula for the computation of the area of a circular figure.
3. Given radius, they will be able to compute the area of a circular figure with utmost accuracy in their respective sq. units.
4. Given area of a circular figure, they will be able to tell about the length of the radius of this circular figure.
5. Given two circular figures of the different diameters or radius, they will be able to tell about the largeness or smallness of these areas.
6. They will be able to apply the knowledge and skills acquired in the computation of the area of a circle in solving day-to-day life problems.

Let us now try to frame the items of the proposed diagnostic tests in view of above cited pre-requisites and expected learning outcomes.

Note. Answer all the question given below at your own. Although there is no time limit yet try to finish your work as early as possible preferably in an hour or so.

1. In the centre of the floor of a big hall there is a circular spot of the radius measuring 3.5 meters. It is to be covered by the gracious Italian tiles costing 1000/ per sq. metre. Tell what one of the following you have to compute for the payment of the cost incurred in installing these tiles.

(a) Circumference of the circular spot

(b) Length and breadth of the circular spot..

(c) Area of the circular spot.

(d) Volume of the circular spot.

2. Write down the formula for the computation of the area of a circular figure.
3. Compute the area of the circular spot (with a radius of 3.5 meters) in the problem 1 mentioned above.
4. Compute the cost involved in flooring the circular spot (with a radius of 3.5 meters) with the tiles in the problems 1 mentioned above.
5. There are two pieces of agricultural land. One is rectangular measuring 80 meter in length and 60 meter in breadth. The other one is circular in shape with a measured radius of 42 meters. Tell which one of them is bigger in size.
6. A person wants to cover 616 sq. meter area of his farmhouse with a circular shaped spot. Tell what will be the radius of this circular shaped spot.
7. Confirm the constancy of the value of 6 with the given data : Area of the circle = 1386 sq. meter

Radius of the circle = 21 meter

Along with the writing of the items for the diagnostic test, attempts should be made to prepare a scoring key and model answers for helping in its proper interpretation.

B. Administration and interpretation of the diagnostic test.

The constructed diagnostic test should now be administered to the class or individual student for knowing about the weaknesses and learning difficulties pertaining to the learning of the sub unit or a single concept. The necessary directions related to the proper administration of the test should here be clearly explained or demonstrated to the students. When they have finished their task, the answer sheets along with the test paper should be carefully collected for the analysis and interpretation of their responses. For this purpose these may be scored on the basis of the scoring key and model answers suggested in the constructed test.

The interpretation about the weaknesses and strength of the students related to their performances in the learning of a sub-unit or single concept then be made on the basis of their scores on the total test items, parts of the test or individual items. A wise step for knowing in depth the nature of learning difficulties and weaknesses of the student lies in carrying out error analysis of the responses given by the group of students or an individual student.

Error Analysis

The term Error analysis as the name suggests stands for the analysis of the error committed by the students in providing responses to the items of the administered diagnostic test for the proper diagnosis of their weaknesses and learning difficulties with the purpose of framing a suitable remedial programme for the removal of these diagnosed weaknesses and learning difficulties.

Error analysis of the diagnostic test in four fundamental rules

S.No.	The error committed	Diagnosed weakness (as a result of the error analysis)
1,	**45** +32 **76**	The student is lacking in the mastery of basic addition combinaions.
9	**67** **-43** **23**	The student is lacking in the mastery of basic subtraction combinations.
3.	**23** **x 3** **66**	The student is lacking in the mastery of basic multiplication combination.
4.	**42 -T- 6 = 6**	The student is lacking in the mastery of basic division combination.
.■ *o** ■	63 +35 88	The student is in the habit of rewriting numerals without computing.
6.	567 x 14 2268	The student multiplies by only one digit out of the two present in the given number.
7.	90 8)721 720	The student ignored remainder because: i. he has not completed subtraction ii. he is not feeling the need for further computation iii. he does not know what to do with "1" if subtraction occurs, so does not compute further.

In this way, through careful study and analysis of the error committed by the students in responding to the various items of a diagnostic test one can reach to some or the other most convincing reasons (in terms of the learner's difficulty and weakness) for the errors committed.

Such type of error analysis and interpretation of the responses of the students **in** terms of their weaknesses and learning difficultis then may lead, a mathematics teacher to think about the possible remedial measures with the sole objective of helping the students to get rid of their weaknesses and learning difficulties. In this way, the findings and results

of the diagnostic tests (coupled with some other types of surveys, results of achievement tests, administration of interest and aptitude tests, attitude scales, case history etc.) may help him to proceed on the path of remedial teaching.

Difference between Achievement and Diagnostic Test

Achievement Tests	Diagnostic Test
The achievement test tells the teacher how much content the individual has learnt.	Diagnostic tesr t. •. how much content still he has to learn to master a specific content.
It never tells how he still has to learn in a specific content. But it evaluates whether not the particular instructional objective is mastered by the student. All types of questions like essay, short answer and objective types are included.	Objective type questions are included.
Achievement tests are meant for the average students.	Diagnostic tests are normally meant for the below average students.
Achievement tests are used to evaluate the achievement level of the students.	Diagnostic tests are used to identify the difficulties and weaknesses of the students.
Achievement tests are used to compare achievement of students and also for grading.	Diagnostic tests lead to remedial teaching or special coaching.
These relate to the entire unit covered.	These concentrate on difficult areas.
These at various occasions determine the effectiveness of the teacher in the teaching learning process.	These are not for evaluating the efficiency of the teaching or the system.

Prognostic Test

The first of these functions is that of prognosis function. Any test tells about some differences among peoples performance at this movement. All decisions involve prediction.

When psychological test is mentioned, so called I.Q. test (intelligence test) administered to students in school to predict their academic performance comes to mind. The measurement provides the extent of a variable which has the specific purpose of predicting future behaviour.

Prognosis means foretelling, prediction and forecasting. The prognostic tests are designed to predict the student's ability or readiness to undertake the study of a school subject successfully.

Uses of Prognostic Test

The test scores gained through prognostic testing indicate not only students present level of achievement, but they also tell or indicate about the possibility of future achievement. Prognostic tests tell the teacher as to what extent his students can derive advantage for further learning experiences. The prognostic testing will indicate whether a student has acquired the mastery of the subject or not if that student wants to take up a higher course in the subject he studied.

Several tests have been developed in India for prognosis purpose. These tests are used generally with children shortly after their entry into the first grade. The objective at this stage is to give the school as accurate an indication as possible of the child's ability to progress in reading. It may be remembered that the reading readiness test is given only to predict ability to profit form reading instruction in the near future. It is not used to forecast ultimate level of learning.

A large number of reading readiness prognostic tests have been developed. Following are the main categories of these tests.

- Ability to read letters or word tests
- Oral vocabulary tests
- Rhyming or matching sounds tests
- Visual matching of figures, letters or word tests.

5.2 CRITERION TEST AND NORM REFERENCED TEST

In order to interpret the marks, marks are referenced to something outside the test. To forms of marks referencing schemes are generally used. They are

1. Criterion test
2. Norm referenced test

Criterion test

It is meant to measure the achievement of an examinee on a certain domain to find out his level of achievement in that domain. It has little to do with the achievement level of other examinees. It relates a student's score on an achievement test to a domain knowledge rather than to another students score.

Its main objective is to measure student's achievement of curriculum based skills. It is prepared for a particular grade or course level. It has balanced representation of goals and objectives. It is used to evaluate the curriculum plan instruction progress and group students' interaction. It can be administered before and after instruction.

- It stresses what examinees can do and what they cannot do.
- It focuses on a delimited domain of learning tasks with a relatively large number of items measuring in each specific task.
- It contains easy and difficult items.
- A student is tested after each unit for mastery of objectives.
- A student is allowed to proceed to the new material if mastery is obtained.
- A student is allowed to go to next unit along with the whole class.
- A student is presented with the new materials of the next unit.
- A student is tested for the new material and assigned marks.

Norm referenced test

A Norm-referenced test is designed to measure individual differences in achievement, intelligence, interest, attitudes or personality. Percentile Ranks, Grade-equivalently scores and standard scores are the examples of norm referenced test. Norm,-referenced test if primarily used for comparing achievement of an examine to that of a large representative group of examinees at the same grade level. The representative group is known as the 'Norm Group'. Norm group may be made up of

examinees at the local level, district level, state level or national level.

Gilbert Sax

A norm referenced test is designed to measure the growth in a student's attainment and to compare his level of attainment with the levels reached by other students and norm group.

Bormuth

Norm referenced test is prepared for a particular grade level. It is administered after instruction. It can be used for forming homogeneous or heterogeneous class groups. It classifies achievement as above average or below average for a given grade.

- ♦ It stresses discrimination among individuals.
- ♦ It covers a large domain of learning task with just a few items measuring each specific task.
- ♦ It contains items of average difficulty.
- ♦ A student is tested after each unit of the new material presented.

♦♦♦ A student is assigned the marks or grades to indicate his performance.

- ♦ A student is given remedial instruction if the material presented is not mastered.
- ♦ A student is tested again after remedial work, to check for mastery of the material.
- ♦ A student is tested for mastery of the already learned unit.

comparison of norm referenced test and criterion referenced test

- ♦ A norm referenced test typically attempts to measure a more general category of competencies.
- ♦ A Criterion referenced test typically focuses on a more specific domain of examinee behaviours.
- ♦ A 100 item on referenced test may be needed to cover the entire range of learners reading comprehension skills as against five separate twenty item criterion referenced test focusing only on five well defined skills within the overall realm of reading comprehension.

CONTINUOUS AND COMPREHENSIVE EVALUATION [CCE]

Philosophical Basis :

The primary purpose of education is the manifestation of perfection already in man and woman [Swami Vivekananda]; purpose of education is all round development of the child / individual. The Report of the International Commission on Education for 21st Century to UNESCO referred to four planes of living of human individuals namely; physical, intellectual, mental and spiritual. Thus, all round development as the stated purpose of education implies optimization of hidden potential of every child in the physical, intellectual, mental and spiritual planes. The CBSE in 2010 initiated for the first time an effort to translate the lofty goal of all round development into practice by introducing CCE - scheme in schools.

Globalisation in every sphere of society has important implications for education. We are witnessing increasing commercialization of education. We need to be vigilant about the pressures to commodity schools and the application of market-related concepts to schools and school quality. The increasingly competitive environment into which schools are being drawn and the aspirations of parents place a tremendous burden of stress and anxiety on children, to the detriment of their personal

growth and development and thus hamper the joy oflearning.

The aims of education simultaneously reflect the current needs and aspirations of a society, its lasting values, concerns as well as broad human ideals. At any given time and place, they can be called the contemporary and contextual articulation of universal human values.

The Frame Work

An understanding of learners, educational aims, the nature of knowledge, and the nature of the school as a social space can help us arrive at principles to guide classroom practices. Conceptual development is thus a continuous process of deepening and enriching connections and acquiring new layers of meaning. Alongside is the development of theories that children have about the natural and social worlds, including themselves in relation to others, which provide them with explanations for why things are the way they are, the relationships between causes and effects, and the bases for decisions and acting. Attitudes, emotions and values are thus an integral part of cognitive development, and are linked to the development of language, mental representations, concepts and reasoning.

As children's metacognitive capabilities develop, they become more aware of their own beliefs and capable of regulating their own learning.

Accordingly, National Curriculum Framework - 2005 [NCF-05] proposing Examination Reforms has stated - "Indeed, Boards should consider, as a long-term measure, making the Class X examination optional, thus permitting students continuing in the same school [and who do not need a Board certificate] to take an internal school examination instead".

As a sequel to the above, the Position Paper on 'Examination Reforms' by NCERT 2006, says, "Indeed, it is our view that the tenth grade exam be made optional forthwith. Tenth-graders who intend continuing in the eleventh grade at the same school and do not need the Board certificate for any immediate purpose, should be free to take a school-conducted exam instead of the Board exam."

Obviously, the efforts of CBSE to provide a leadership and pioneering role in implementing CCE is a major breakthrough which attempts to elevate the status of the schools as equal partners of the Board in assessing the attainment levels of learners.

Place of Evaluation in the Curriculum

A curriculum is what constitutes a total teaching-learning program composed of overall aims, syllabus, materials, methods and assessment. In short it provides a framework of knowledge and capabilities, seen as appropriate to a particular level. The syllabus provides a statement of purpose, means and standards against which one can check the effectiveness of the program and the progress made by the learners. Evaluation not only measures the progress and achievement of the learners but also the effectiveness of the teaching materials and methods used for transaction. Hence evaluation should be viewed as a component of curriculum with the twin purpose of effective delivery and further improvement in the teaching-learning process.

What is Continuous and Comprehensive Evaluation?

Continuous and Comprehensive Evaluation [CCE] refers to a system of school-based assessment of students that covers all aspects of students' development.

It is a developmental process of assessment which emphasizes on two fold objectives. Continuity in evaluation and assessment of broad based learning and behavioural outcomes.

In this scheme the term 'continuous' is meant to emphasise that evaluation of identified aspects of students' 'growth and development' is a continuous process rather than an event, built into the total teaching-learning process and spread over the entire span of academic session.

The second term 'comprehensive' means that the scheme attempts to cover both the Scholastic and the Co-Scholastic aspects of students' growth and development. Since abilities, attitudes and aptitudes can manifest themselves in forms other than the written word, the term refers to application of a variety of tools and techniques and aims at assessing a learner's development in higher order thinking skills such as analyzing, evaluating and creating. Assessment during the course of studies or formative assessment must be based on a variety of evidences and lead to diagnosis of learning gaps and their remediation.

The scheme is thus a curricular initiative, attempting to shift emphasis from memorizing to holistic learning. It aims at creating citizens possessing sound values, appropriate skills and desirable qualities besides academic excellence. It is hoped that this will equip the learners to meet the challenges of life with confidence and success. It is the task of school based co-scholastic assessment to focus on holistic development that will lead to lifelong learning.

The objectives of the CCE scheme are

- To help develop cognitive, psychomotor and affective skills.
- To lay emphasis on thought process and de-emphasise memorization.
- To make evaluation an integral part of teaching-learning process.
- To use evaluation for improvement of students' achievement and teaching- learning strategies on the basis of regular diagnosis followed by remedial measures.
- To use evaluation as a quality control devise to raise standards of performance.
- To determine social utility, desirability or effectiveness of a programme and take appropriate decisions about the learner, the process of learning and the learning environment.
- To make the process of teaching and learning a learner-centered activity.

Features of Continuous and Comprehensive Evaluation

- The 'continuous' aspect of CCE takes care of the 'continual' and 'periodicity' aspect of evaluation.
- Continual means assessment of students in the beginning of instructions [placement evaluation] and assessment during the instructional process [formative evaluation], done informally using multiple techniques of evaluation.
- Periodicity means the assessment of performance done at the end of a unit/term [summative].
- The 'comprehensive' component of CCE takes care of assessment of the all round development of the child's personality. It includes assessment of Scholastic as well as Co-Scholastic aspects of the pupil's growth.
- Scholastic aspects include curricular areas or subject specific areas, whereas Co-Scholastic aspects include Life Skills, Co-Curricular activities, attitudes and values.
- Assessment in Scholastic areas is done informally and formally using multiple techniques of evaluation continually and periodically. The diagnostic evaluation takes place at the end of

unit/term test. The causes of poor performance in some units are diagnosed using diagnostic tests. These are followed up with appropriate interventions and remedy measures.

- Assessment in Co-Scholastic areas is done using multiple techniques on the basis of identified criteria; where assessment in Life Skills is done on the basis of Indicators.

FORMATIVE AND SUMMATIVE ASSESSMENTS

Formative Assessment [FA] is a tool used by the teacher to continuously monitor a student's progress in a non-threatening, supportive environment. It involves regular descriptive feedback, a chance for the student to reflect on the performance, take advice and improve upon it. If used effectively, it can improve student performance tremendously, while raising the self-esteem of the child and reducing the work load of the teacher.

Salient features of Formative Assessment:

- diagnostic and remedial.
- makes the provision for effective feedback.
- provides the platform for the active involvement of students in their own learning.
- enables teachers to adjust teaching to take account of the results of assessment.
- recognizes the profound influence assessment has on the motivation and self-esteem of students, both of which are crucial and influences learning.
- recognizes the need for students to be able to assess themselves and understand how to improve.
- builds on students' prior knowledge and experience in designing what is taught.
- incorporates varied learning styles into deciding how and what to teach.
- encourages students to understand the criteria that will be used to judge their work.
- offers an opportunity to students to improve their work after feedback.
- helps students to support their peers, and expect to be supported by them.

Formative Assessment is thus carried out during a course of instruction to provide continuous feedback to both the teachers and the learners, to take decisions regarding appropriate modifications in the transactional procedures and learning activities.

It involves students' being an essential part of assessment. They must be involved in a range of activities right from designing criteria to self assessment or peer assessment. Summative Assessment [SA] is carried out at the end of a course of learning. It measures or 'sums-up' how much a student has learned from the course. It is usually a graded test, i.e., it is marked according to a scale or set of grades.

Assessment that is predominantly of summative nature will not by itself be able to illustrate a valid measure of the growth and development of the child. It, at best, certifies the level of achievement only at a given point of time. The paper-pencil tests are basically a onetime mode of assessment; and to exclusively rely on it to decide about the development of a child is not only unfair but also unscientific. Overemphasis on examination marks focusing on only scholastic aspects makes children assume that assessment is different from learning, resulting in the 'learn and forget' syndrome. Besides encouraging unhealthy competition, the overemphasis on Summative Assessment system also produces enormous stress and anxiety among learners. It is this that has led to the emergence of the concept of Continuous

and Comprehensive School-Based Evaluation.

Formative Assessment [b]

The marking and grading of FA[b] activities would be based on the assessment of written work.

A few types of questions /tests for FA [b] arc suggested as under:

Right/Wrong, True-false, Yes/No. Matching Type, Matrix Type, Multiple Choice Questions, Choose the correct answer, Fill in the blanks, Completing Railway reservations forms and bank challans etc., Sequencing questions, Dictation, Very Short Answers, Short Answers, Filling Summary Formats/ Graphic Organizers, Riddles, Puzzles and Competency-related Drawing. For differently-abled children, or children who have been recently mainstreamed, other appropriate modes of assessment may be evolved and used.

Recording FA [b] Activities

As soon as the FA [b] tests are completed in each term, learners are to be facilitated to record the completion of FA [b] in the Student Learning Activity Record [Annexure -1].

The teachers need to meticulously evaluate the answer scripts of learners and award marks to them according to the correctness of the answer. The mark secured by each student for each subject in all the four tests is to be recorded in the Teacher Assessment Record [Annexure - II].

Out of a minimum of four tests conducted for learners, two tests that reflect the best performance may be chosen for marking and grading.

The corresponding grade for the marks secured by each student is to be entered in the Student Cumulative Record. If the mark secured by the students is 5 or less than 5, they need to be given special attention and remedial activities

Guidance for summative Assessment

- The focus of marking and grading will be on written work.
- This will be a normative pen-and-paper Examination given at the end of each them.
- The Summative Assessment will be based on the prescribed blueprint.
- The questions for summative assessment may be asked not only from the exercise given at the end of lesson but teachers can also ask questions related to higher order thinking skills.

GRADING PATTERN

Table - 1

Formative Assessment FA 40		Summative Assessment SA 60		Total FA + SA 40+60=60	
Marks	Grade	Marks	Grade	Marks	Grade
37-40	Al	55-60	Al	91-100	Al
'33-36	A2	49-54	A2	81-90	A2
29-32	Bl	43-48	Bl	71-80	Bl
25-28	B2	37-42	B2	61-70	B2
21-24	CI	31-36	CI	51-60	CI
17-20	C2	25-30	C2	41-50	C2
13-16	D	19-24	D	33-40	D

9-12	El	13-18	El	21-32	El
8& Below	E2	12 & Below	E2	20 & Below	E2

Table - TI

Grade	Grade Point	Explanation for the Grade
A	**5**	Excellent [If all the five indicators are exhibited]
B	**4**	Very Good [If any four indicators are exhibited]
C	**3**	Good [If any three indicators are exhibited
D	**2**	Satisfactory [If any two indicators are exhibited]
E	**1**	To be Strengthened [If any one indicators are

Table - III

Average	
Grade Point	Grade
4,5-5	A
3.5-4.4	B
2.5-3.4	C
1.5-2.4	D
1.0-1.4	E

5.3 STATISTICAL MEASURES

As much as teaching and testing go together, statistics and testing go hand in hand. Through tests, we can find out

(1) how much pupils learn from our teaching, i.e the achievement of our pupils, and

(2) whether our teaching is effective or not.

This can be done by treating and processing the test results.

Statistics, as we know, is the study of the collecting and analyzing of data to see what inference and conclusions can be drawn.

Statistics can be defined as the legal data, which describe, summarize, generalize the condition of a country or a school or a department or a student. It is essential and indispensable in important fields of specialization in education.

After a test has been given, it is essential to arrange the scores in some order and do some calculations to interpret the results. The basic knowledge of statistics is essential to enable us to process these test scores.

The aim of teaching statistics in this course is to enable the students to

(1) construct test systematically,

(2) compile and analyze data, and

(3) interpret the results scientifically.

Frequency Distribution

The score by itself is meaningless. They are only useful, if order can be put into the collected scores. It is the statistical methods that put orders into data.

One of the most fundamental techniques for putting order into a disarray of data is the frequency distribution. Basically, it is a systematic procedure for arranging individuals from least to most in relation to some quantifiable characteristics.

Frequency distributions are constructed primarily for two reasons. First, they put the data into order so that visual analysis can be made of the results of the measurements, and secondly, they provide a convenient structure for simple computations.

Frequency distributions mean the way the scores within a group are spread out. From the frequency distribution, we can know

(1) the number of scores above and below the Mean.

(2) whether me scores are all bunched up around the Mean or spread about quite far away form the Mean towards either end of the range.

The two types of Frequency distribution tables are as follows:

(1) Frequency distribution table where actual scores are used as groups, and

(2) Frequency distribution table where ordered data are grouped within the range of intervals.

Frequency distribution table can be set up by

(1) finding the range,

(2) dividing the difference by 15 or 10

(3) writing down the class intervals, and

(4) taking frequency counts.

An example using English test scores of 25 students will illustrate the procedure for setting up the two types of frequency distribution table.

English test scores of 25 students

75	72	77	68	71
80	78	63	87	72
67	70	66	70	72
82	83	76	76	70
62	65	84	68	62

Table 1 A Frequency Distribution of English Test Scores of 25 Students

Scores X	Number of Student (frequency)	Scores X	Number of Student (frequency)
87	1	74	0
86	0	73	0
85	0	72	3
84	1	71	1
83	1	70	3
82	1	69	0
81	0	68	2
80	1	67	1
79	0	66	1
78	1	65	1
77	1	64	0
76	2	63	1
75	1	62	2

Table 2 A Second Frequency Distribution of English Test Scores of 25 Students

Class Interval *X*	Frequency *f*
85-87	1
82-84	3
79-81	1
76-78	4
73-75	1
70-72	7
67-69	3
64-66	2
61-63	3
Total	25

From the above frequency distribution, we can calculate the followings:

(1) The Range

Range = Highest Score – Lowest Score

From Table 1, Range = 87 – 62 = 25

From Table 2, Range = 86 – 62 = 24

(2) Exact Limits of The Class Interval (85-87)

Exact Lower Limit = 84.5

Exact Upper Limit = 87.5

(3) Size of class interval (CI)

CI = U – L

CI for the interval (85-87) = 87.5 – 84.5 = 3

(4) Mid point of the class interval $x = \frac{(U + L)}{2}$

Mid point of the class interval (85-87) = $\frac{(84.5 + 87.5)}{2} = 86$

(5) $CI = \frac{\text{Range}}{\text{No. of Class Intervals}}$

(i) If range is 60, and number of class intervals is 12, then CI equals 5.

(ii) If range is 80, and CI is 5, then the number of class interval equals 16

(iii) If the number of class intervals is 10 and CI is 3, then range equals 30

Exercises

(1) Find the exact limits, mid point and CI of the interval (56-60).

(2) Given the following IQ data, construct a frequency distribution according to prescribed procedures.

144, 116, 97, 111, 112, 85, 132, 128, 123, 106, 80, 93, 118, 113, 104, 121, 101, 117, 138, 122, 118, 112, 109, 114, 105, 125, 129, 133, 103, 92.

(3) Using the given data, construct two frequency distribution table assuming CI = 3, and CI = 5.

115, 108, 102, 106, 103, 97, 104, 105, 110, 97, 101, 103, 92, 92, 105, 104, 106, 93, 103, 95, 104, 98, 106, 91, 102, 99, 103, 96, 99, 102.

Graphic Representation of Data

Graphs are used in the practical handling of real set of data. It is also used as visual models in thinking about statistical problems.

The graphic representation of education of educational data takes various forms. They are:

(1) Pie Chart

(2) Bar Graph

(3) Pyramid

(4) Histogram

(5) Frequency Polygon

(6) Cumulative Frequency Curve

(7) Cumulative Percentage Curve or Ogive

(1) Pie Chart

The pie chart is a convenient way of indicating the various components of a whole. It is useful when one wishes of picture proportions of the total in a striking way.

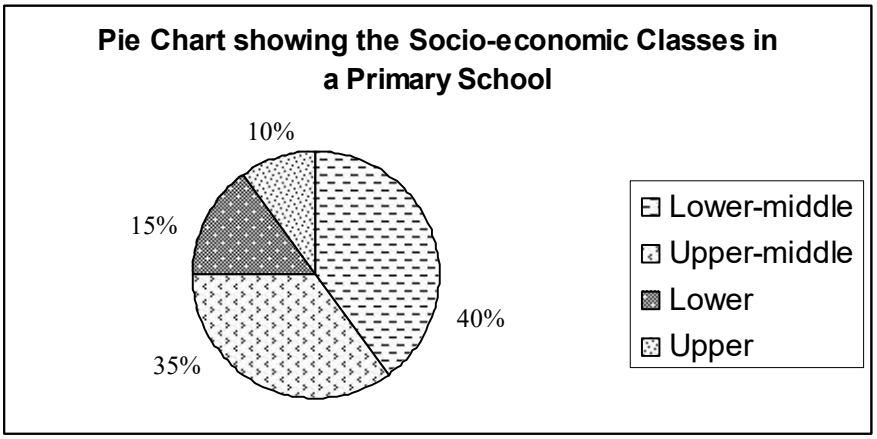


Fig 1 Pie Chart showing the socio-economic classes in a Primary School

(2) Bar Graph

Bar graphs are commonly used to represent the size of measures, scores, percentages etc. It is often used in Education and Psychology to compare the relative amounts of some traits (height. intelligence) possessed by two or more groups.

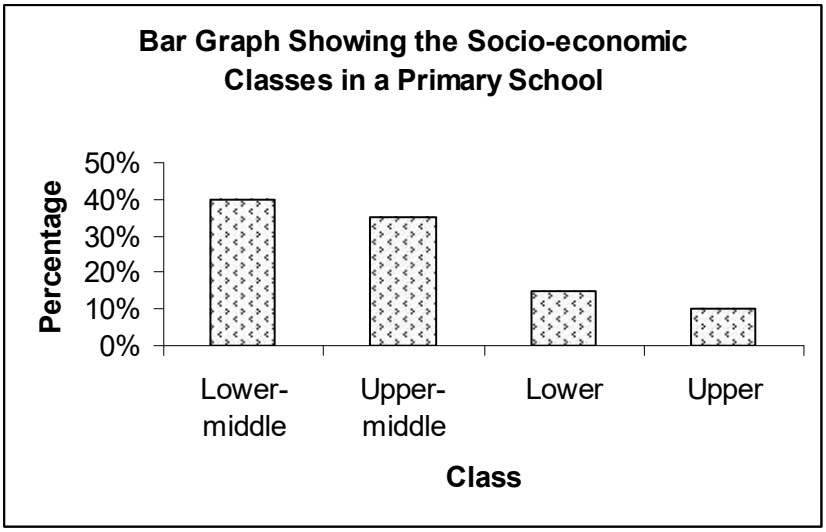


Fig 2 Bar graph showing the socio-economic classes in a Primary School

(3) Pyramid

Pyramids are used for the purpose of comparing the number of students enrolled by levels, by grades and by sex etc.

The data represented by the pyramid are very helpful for educational administrators and planners.

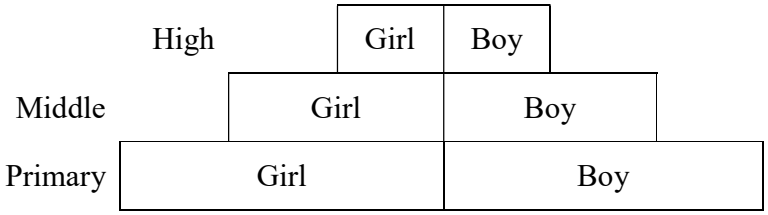


Fig 3 Pyramid showing the general feature of enrollment in a High School.

(4) Histogram

One way of presenting frequency distribution in a graphic form is a histogram. A histogram is a graph plotted by exact limits and their respective frequencies. In a histogram, the scores are assumed to be spread uniformly over the entire interval.

It shows how the scores in the group are distributed-whether they are piled up at the low or high end of the scale or are evenly distributed over the scales.

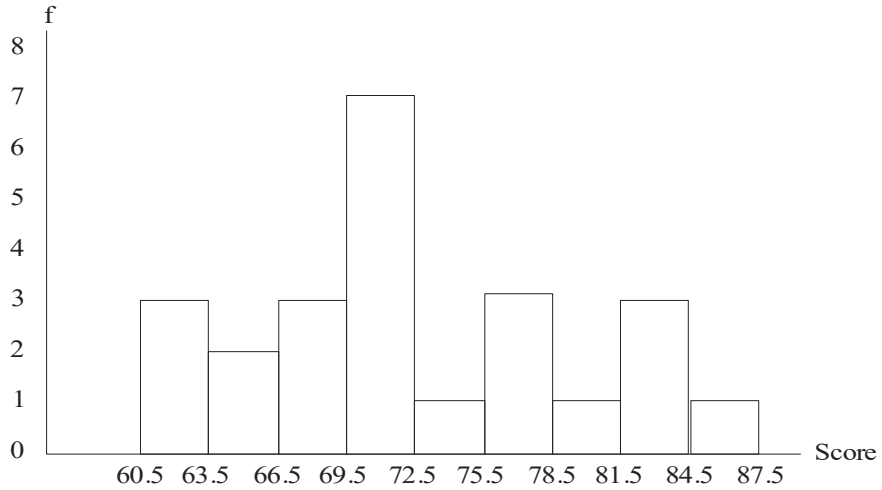


Fig 4 Histogram showing distribution of English test scores of 25 students.

(5) Frequency Polygon

Sometimes we wish to reflect the continuous nature of our data. By placing a point at the center of the top of each bar (class interval) and connecting these points and lines, we will get a frequency polygon. In a frequency polygon, we assume that all cases in each interval are concentrated at the mid-point of the interval. Thus, a frequency polygon is a graph plotted by actual scores or mid points and their respective frequencies. It is likely to be more useful in comparing two or more graphs plotted on the same axes.

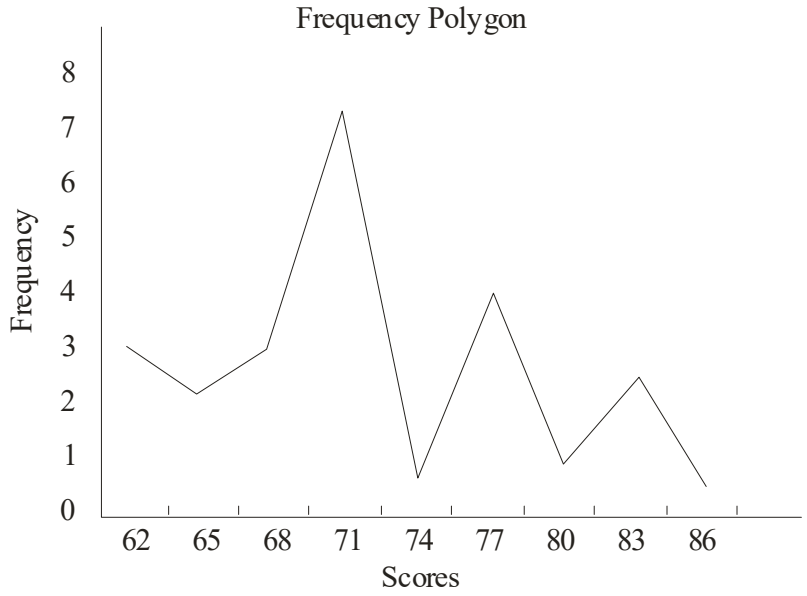


Fig 5 Frequency Polygon showing distribution of English Test Scores or 25 students.

Frequency distribution curve can generally be classified into two types, depending on the shape of the curve.

(1) Symmetrical Curve

(2) Skewed Curve

- Positively Skewed Curve
- Negatively Skewed Curve

(1) Symmetrical Curve

A curve is symmetrical when one half of the curve is a mirror image of the other half.

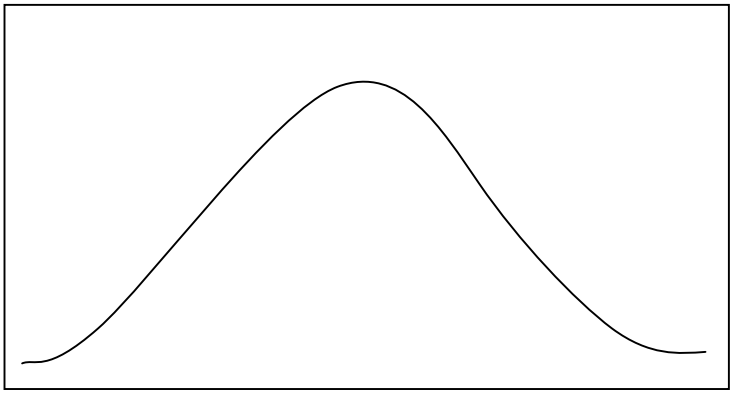


Fig 6 An example of a symmetrical curve

(2) Skewed Curve

A curve is skewed it is not symmetrical.

(i)Positively Skewed Curve

If the massing of the scores is at the left end of the curve with the tail extending to right end, then the curve is positively skewed.

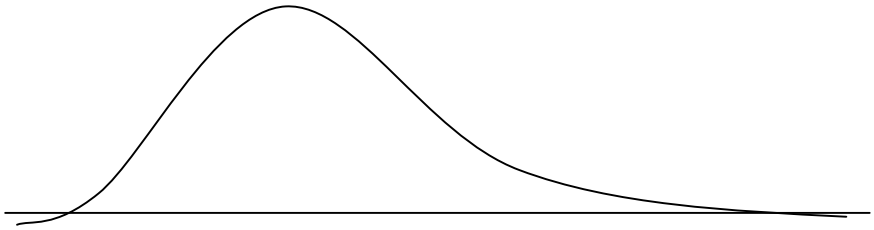


Fig 7 An example of positively skewed curve

(ii)Negatively Skewed Curve

If the massing of the scores is at the right end of the curve with tail extending to the left end, then the curve is negatively skewed.

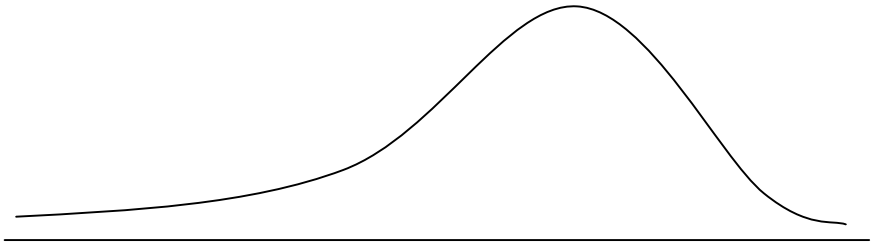


Fig 8 An example of negatively skewed curve

(6)Cumulative Frequency Curve

The cumulative frequency is used to determine the number of cases falling above or below particular values. In cumulative frequency curve, we plot points above the top of the exact limits of the interval.

X	*f*	F	F%
85-87	1	25	100
82-84	3	24	96
79-81	1	21	84
76-78	4	20	80
73-75	1	16	64
70-72	7	15	60
67-69	3	8	32
64-66	2	5	20
61-63	3	3	12

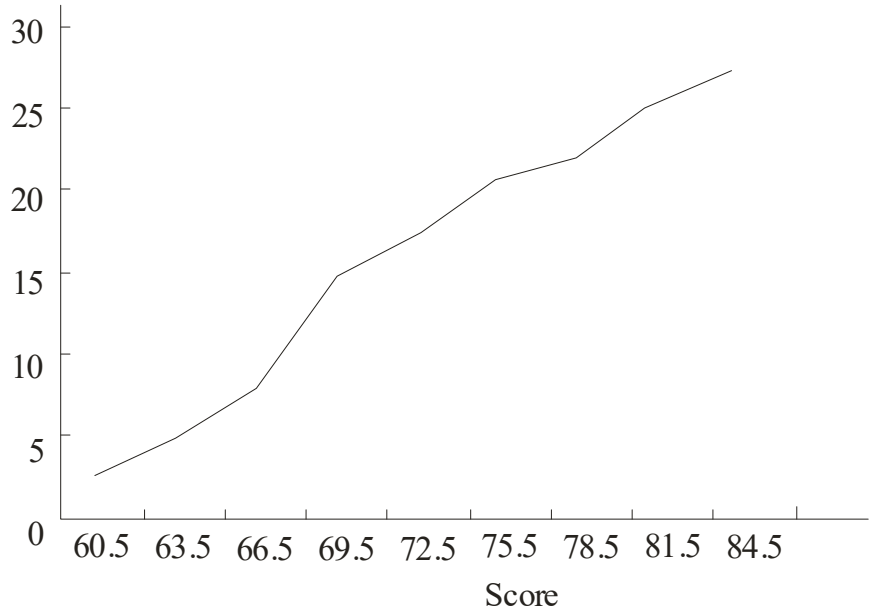


Fig 9 Cumulative frequency curve showing distribution of English test scores of 25 students

(7) Cumulative Percentage Curve or Ogive

We may determine the cumulative percentage frequencies by converting raw frequencies to percentage. A cumulative percentage or Ogive can be obtained by graphing these frequencies.

The advantage of this type of graph is that, from it, we can read off directly the percentage of observations less than any specified values. Percentiles and percentile rank can be determined quickly an fairly accurately from this Ogive.

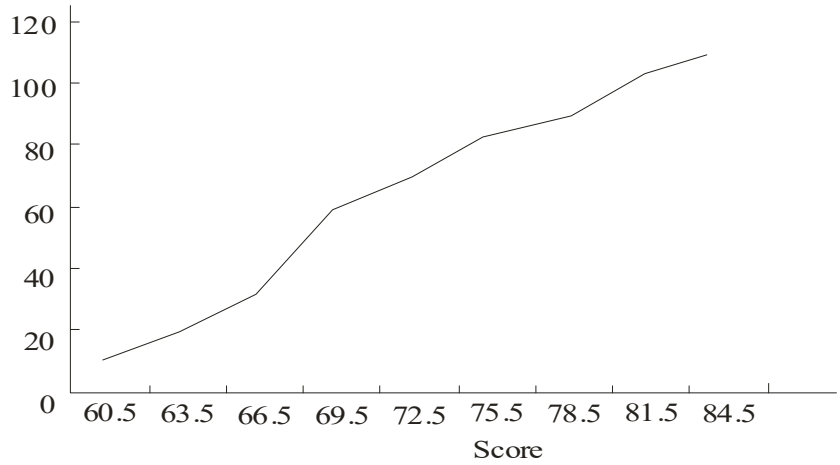


Fig 10 Cumulative Percentage Curve or Ogive showing distribution of English test scores of 25 students

Exercises

(1) Draw a (i) Pie Chart (ii) Bar Chart from the given percentages of students attending school in a township.

Primary Level	45%
Secondary Level	35%
High School Level	15%
Colleges Level	5%

(2) Plot a (i) Histogram (ii) Frequency Polygon (iii) Cumulative Frequency Curve (iv) Cumulative Percentage Curve or Ogive, from the given distribution of spelling test scores.

X	*f*
46-50	4
41-45	6
36-40	10
31-35	18
26-30	6
21-25	3
16-20	1
Total	50

Measures of Central Tendency

In the previous chapters, we are concerned with the organization of collections of numbers in the form a frequency distribution and how such frequency distribution could be presented in tabular or graphic form. Now we will proceed to a consideration of how a collection of numbers may be described in a single value.

Measures of central tendency is a value around which a distribution tends to center. It gives us a concise description of the typical performance of the group as a whole. It enables us to compare the performance of two or more groups. The indicators of central tendency are the mean, the median and the mode.

The Mean

The mean is the arithmetic average. Of the three statistics, it is the most stable from sample to sample.

The mean is simply the sum to series of measures divided by the number of measures. It is represented by the Symbol *X*.

The formula for mean is:

$$\overline{X} = \frac{\sum X}{N}$$

where $\overline{X}$ = mean

X = individual score

N = total number of measures

$\sum$ = summation

Example

(1) Find the mean of the following scores.

19, 20, 26, 18, 21, 8, 20, 24, 15, 18, 20

$\sum X = 209 \quad N = 11$

$$\therefore \overline{X} = \frac{\sum X}{N} = \frac{209}{11} = 19$$

$$\overline{X} = 19$$

The Median

By simple definition, the median is the mid-point in a set of ranked scores.

The median is not influenced by how far extreme scores may range in a given distribution because the range of these scores does not change the point that divides the distribution into two equal-sized groups.

Examples

(1) What is the median of the following set of scores?

22, 28, 19, 25, 8, 23, 20

First, arrange the scores in order of magnitude.

28, 25, 23, 22, 20, 19, 8

Here the middle score is 22.

 The median of this set of scores is 22. There is only one score in the center when working with odd number of scores.

When even number of scores are given, the arithmetic mean of two middle scores would be the median.

(2) What is the median of the following set of scores?

20, 19, 22, 17, 15, 25, 12, 18

First arrange the scores in order of magnitude.

25, 22, 20, 19, 18, 17, 15, 12

 Median = $\frac{18+19}{2} = 18.5$

The Mode

The mode is simply the most frequently occurring score in a distribution. It is less frequently used measure of central tendency. It is used in preference to either the median or the mean, when a measure of the most characteristic of a group is desired.

Example

(1) Find the mode of the given set of scores.

2, 3, 3, 2, 4, 5, 2

Here the most frequently occurring score 2 is the mode.

Mode = 2

(2) Find the mode of the given set of scores.

3, 4, 5, 6, 7, 8

Since all the frequency of each score is equal, there is no mode.

Also if the given scores are 2, 2, 3, 3, 4, 4, 5, 5, there is no mode since the frequency of each score is equal.

(3) Find the mode of the given set of scores.

4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 8, 9

Since the most frequently occurring scores 6 and 7 are consecutive,

$$\text{Mode} = \frac{6+7}{2} = 6.5$$

(4) Find the mode of the given set of scores.

4, 4, 4, 5, 5, 6, 6, 7, 7, 7

Here, there are two modes namely 4 and 7.

Calculation of Measures of Central Tendency from a Frequency Distribution Table

Calculation of the Mean

Example (1) Find the mean using the given frequency distribution table.

Table 1 : Table showing mathematics test scores of 50 students.

X	f	fX
10	2	20
9	4	36
8	9	72
7	15	105
6	8	48
5	7	35
4	5	20
Total	50	336

Here $\sum fX = 336$, $\sum f = 50$

Using the formula $\overline{X} = \frac{\sum fX}{\sum f}$

$\overline{X} = \frac{336}{50} = 6.72$

$\therefore$ Mean $= 6.72$

Steps

1. Multiply each score by its respective frequency.
2. Add these products.
3. Divide the sum of the products by the sum of frequencies.

Example (2) Find the mean using the given frequency distribution table.

Table 2 Table showing English test scores of 50 students.

X	*f*	*X*	*fx*
65-69	2	67	134
60-64	10	62	620
55-59	11	57	627
50-54	12	52	624
45-49	9	47	423
40-44	5	42	210
35-39	1	37	37
Total	50		2675

Here $\sum fX = 2675$, $\sum f = 50$

Using the formula $\overline{X} = \frac{\sum fX}{\sum f}$

$\overline{X} = \frac{2675}{50}$

$= 53.5$

$\therefore$ Mean $= 53.5$

Steps

1. Calculate the mid-point of all intervals.
2. Multiply each mid-point by the corresponding frequencies.
3. Sum the product of mid-point by frequencies.
4. Divide this by sum of frequencies, i.e., (*N*)

Calculation of the Mean by the short method

Example (1) Find the mean from the following distribution table.

X	f	d	fd
10	2	3	6
9	4	2	8
8	9	1	9
7	15	0	0
6	8	-1	-8
5	7	-2	-14
4	5	-3	-15
Total	50		-14

The formula for using the shorts method is

$$\overline{X} = AM + \frac{\sum fd}{\sum f} \text{ x CI}$$

where, AM = Assumed Mean

$$d = \frac{X - AM}{CI}$$

From the table

AM = 7

□□fd = -14

□□f = 50

CI = 1

$$\square\ \overline{X} = 7 + \frac{-14}{50} \text{ x } 1$$

$$= 7 - 0.28$$

$$= 6.72$$

Example (2) Find the mean from the following distribution table.

X	f	d	fd
65-69	2	3	6
60-64	10	2	20
55-59	11	1	11
50-54	12	0	0
45-49	9	-1	-9
40-44	5	-2	-10
35-39	1	-3	-3
Total	50		15

Here AM = 52

$\square\square$fd = 15

$\square\square$f = 50

CI = 5

$$\overline{X} = \text{AM} + \frac{\sum \text{fd}}{\sum \text{f}} \text{ x CI}$$

$$\overline{X} = 52 + \frac{15}{50} \text{ x } 5$$

$$= 52 + 1.5$$

$$= 53.5$$

Combined Mean

Sometimes we have means or several different samples and would like to know the mean for the total combined group.

The formula for computing a combined mean is

$$\overline{X} = \frac{N_1\overline{X}_1 + N_2\overline{X}_2 + \ldots\ldots + N_K\overline{X}_K}{N_1 + N_2 + \ldots\ldots + N_K}$$

Where $\overline{X}$ = mean or combined groups

$N_1, N_2, \ldots\ldots, N_K$ = numbers of cases in sample 1, 2 and k

$\overline{X}_1, \overline{X}_2, ..., \overline{X}_K$ = means of sample 1, 2, K

Example

The mean IQs for there sections of eight standard students are given.
Find the combined means for the eighth standard.

	$\overline{X}$	N
Section 1	101	30
Section 2	107	35
Section 3	95	26

Using the formula

$$\text{Combined mean } \overline{X} = \frac{N_1\overline{X}_1 + N_2\overline{X}_2 + ... + N_K\overline{X}_K}{N_1 + N_2 + ... + N_K}$$

$$= \frac{30(101) + 35(107) + 26(95)}{30 + 35 + 26}$$

$$= 101.6$$

Calculation of the Median

Example : Calculate the median for the following distribution table.

X	f	F
65-69	2	50
60-64	10	48
55-59	11	38
50-54	12	27
45-49	9	15
40-44	5	6
35-39	1	1
Total	50	

In calculating the Median, we must first set up a cumulative frequency column. The formula for calculating the Median is

$$\text{Mdn} = L + \frac{\frac{N}{2} - F}{f} \text{ x CI}$$

where, L = exact lower limit of the interval containing the median

F = cumulative frequency below the class containing the median

CI = class interval

In this example $\frac{N}{2} = \frac{50}{2} = 25,\ L = 49.5,\ F = 15,\ f = 12$

$$\text{Mdn} = L + \frac{\frac{N}{2} - F}{f} \text{ x CI}$$

$$= 49.5 + \frac{25 - 15}{12} \text{x } 5$$

$$= 49.5 + 4.166$$

$$= 53.666$$

Mdn = 53.67

Calculation of the Mode

Example : Calculate the mode for the following distribution table.

X	f
65-69	2
60-64	10
55-59	11
50-54	12
45-49	9
40-44	5
35-39	1
Total	50

The formula for calculating the mode is

$$M_0 = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \text{ x CI}$$

where M_0 = Mode

L = exact lower limit containing mode

Δ_1 = $f_0 - f_{-1}$

Δ_2 = $f_0 - f_{+1}$

CI = Class Interval

In this example

L = 49.5

Δ_1 = 12 – 9 = 3

Δ_2 = 12 – 11 = 1

CI = 5

$$M_0 = 49.5 + \frac{3}{3+1} \times 5$$

$$= 49.5 + \frac{15}{4}$$

$$= 49.5 + 3.75$$

$$= 53.25$$

Comparison of the Mean, Median and Mode

If the frequency distribution is Symmetrical, the mean, median and mode coincide.

If the frequency distribution is skewed, these three measures do not coincide. (Fig 1)

If the mean is greater than the median, then the distribution is positively skewed. (Fig 2)

If the mean is less than the median, then the distribution is negatively skewed. (Fig 3)

Thus just by knowing the values of mean, median and mode, the shape of the distribution can be easily calculated.

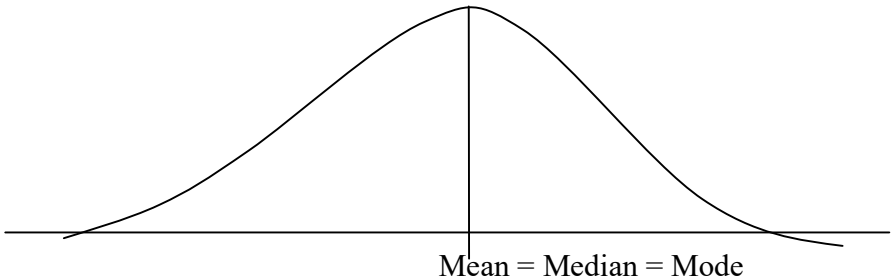


Fig (1) A Symmetrical Curve

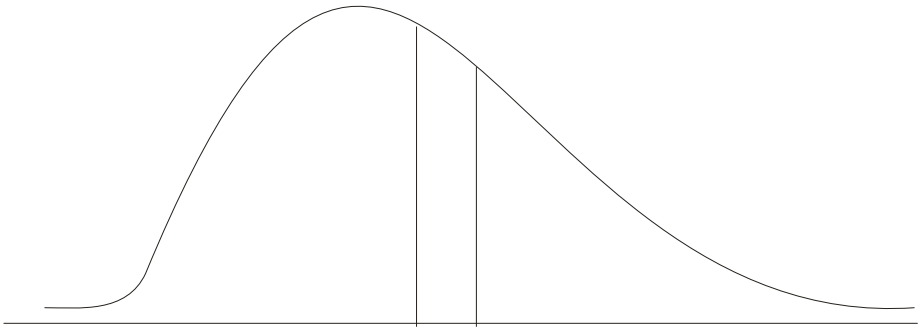


Fig (2) A Positively Skewed Curve

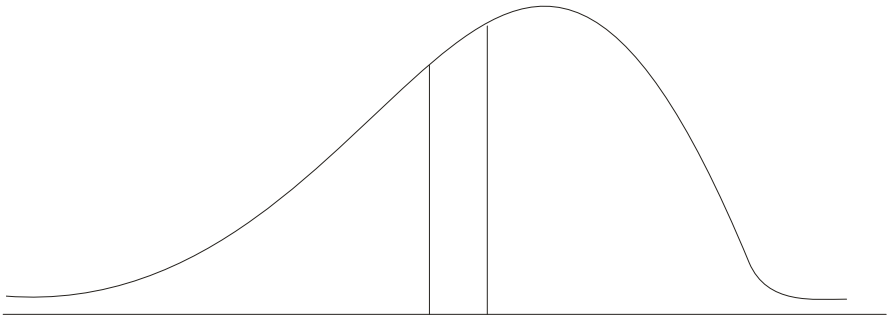


Fig (3) A Negatively Skewed Curve

Exercises

1. Compute means, medians and modes for the following data.
   (a) 3, 8, 18, 36, 54
   (b) 11, 20, 19, 29, 29, 45
   (c) 3, 3, 4, 6, 6, 6, 7, 7, 7, 9

2. Calculate the mean, median, and mode for the following distribution and trace the shape of it.

X	f
85-89	4
80-84	5
75-79	5
70-74	20
65-69	18
60-64	10
55-59	13
50-54	10
45-49	4
40-44	6
35-39	5
Total	100

Measures of Variability

Statisticians have taken advantage of two outstanding features of most frequency distributions:

(1) The tendency of the data, to cluster around some value lying between the smallest and largest data.

(2) The tendency of the data to be dispersed around this central value.

The first feature is **central tendency** and the second feature is dispersion or **variability.**

Although measures of central tendency are very useful statistics for describing a set of data, they do not tell us enough. Two sets of data, which are very different, can have identical means, or medians. As an example, consider the following sets of data.

Set A	59	59	59	60	61	61	61
Set B	30	40	50	60	70	80	90

The mean of both sets of score is 60 and the median of both is 60, but set A is very different from set B. In set A, the scores are very close together and clustered around the mean. In set B, the scores are much more spread out; that is, there is much more variation or variability in set B. Thus, there is a need for a measure that indicates how spreads out the scores are and how much variability there is.

Measures that indicate the amount of scatter in a distribution scores are referred to as measures of variability. In other words, measures which reflect the way in which data are spread in either direction from the center of a distribution are called measures of variability or dispersion or spread ness or scatter.

Five measures have been devised to indicate the variability or dispersion within a set of data. They are :

(1) The range

(2) The quartile deviation

(3) The mean deviation

(4) The standard deviation

(5) The variance

The Range

The range is defined either as the difference between the highest score and the lowest score (R = H – L) or as the difference Plus one (R = H – L + 1), the latter being more accurate. For example the range for the scores 59, 59, 59, 60, 61, 61, 61 is 3 where as the range for the scores 30, 40, 50, 60, 70, 80, 90 is 61. Thus, if the range is small, the scores are close together whereas the range is large, the scores are more spread out. Like the mode, the range not a very stable measure of variability and its chief advantage is that it gives a quick, rough estimate of variability.

The Quartile Deviation

The quartile deviation (also referred as the semi-inter quartile range) is one-half of the difference between the upper quartile and the lower quartile in a distribution. The upper quartile is the 75^{th} percentile that point below which are 75% of the scores, the lower quartile. Correspondingly, is the 25^{th} percentile, that point below which are 25% of the scores. By subtracting the lower quartile from the upper quartile then dividing the result by two, we get a measure of variability.

$$QD = \frac{Q_3 - Q_1}{2}$$

As an example, if there are 60 scores, Q_1 is the point below which are 15 of the scores (15 = 25% of 60), and Q_3 is the point below which are 45 of the score (45 = 75% of 60). If the quartile deviation is small, the scores are together whereas if the quartile deviation is large, the scores are more spread out. The quartile deviation is a more stable measure of variability than the range and is appropriate whenever the median is appropriate. Calculation of the quartile deviation involves a process very similar to that used to calculate the median, which just happens to be the second quartile Q_2.

The Mean Deviation

Unlike the range and the quartile deviation, the mean deviation takes into account every score in the distribution. The difference, or deviation, or each score from the mean of the distribution is determined without regard to the direction of the difference. These deviations are added and the sum

is divided by the number of scores. The result is the mean of the deviations of the scores from their mean.

The mean deviation does not have a wide range of application, nor it is very useful for making comparisons among sets of data. The main reason is that the absolute values of the deviations of scores the mean are used in the computation. In other words, we ignore the signs of deviations.

The Standard Deviation

The standard deviation is appropriate when the data represent an interval or ratio scale and is by far the most frequently used index of variability. Like the mean, the measure of central tendency which is its counterpart, the standard deviation is the most stable measure of variability and takes into account each and every score. In fact, the first step in calculating the standard deviation involves finding out how far each score is from the mean, that is, subtracting the mean from each score. If we square each difference, add up all the squares, and divide by the number of scores, then we have a measure of variability called variance. The square root of that measure is called the standard deviation. A small standard deviation indicates that scores are close together and a large standard deviation indicates that the scores are more spread out.

If you know the mean and the standard deviation of a set of scores, you have a pretty good picture of what the distribution looks alike. If the distribution is relatively normal, then the mean plus 3 standard deviations and the mean minus 3 standard deviations encompasses just about all the scores over 99% of them.

For example, suppose that for a set of scores the mean ($\overline{X}$) is calculated to be 60 and the standard deviation (SD) to be 1. In this case, the mean plus three standard deviations, $\overline{X}$ + 3SD is equal to 60 + 3(1) = 60 + 3 = 63. The mean minus three standard deviations, $\overline{X}$ - 3SD is equal to 60 – 3 = 57. The scores fall between 57 and 63. This makes the scores to be close together or not to be very spread out.

As another example, suppose that for another set of scores the mean (X) is again calculated to be 60, but this time the standard deviation (SD) is calculated to be 5. In this case, the mean plus three standard deviations and the mean minus three standard deviations are 75 and 45 respectively. Thus, almost all the scores between 45 and 75. This almost makes sense since a larger standard deviation indicates that the scores are more spread out.

The Variance

The variance is nothing but the square of the standard deviation. Thus, in order to determine the variance of a distribution, each of the deviation scores is squared. These squared deviations are then added together and divided by the number of scores in the distribution. The mean of sum of these square deviations is called the variance of the distribution.

If the variance is small, the scores are close together, if the variance is large, the scores are more spread out. Both variance and standard deviation are most widely used statistical indices of variability and are fitted into further statistical analyses.

Calculation of Measures of Variability from Ungrouped Data.

Example (1) Find range, quartile deviation, standard deviation and variance of the following data.

10, 15, 13, 8, 10, 10, 15, 13, 11, 7, 4, 4

(1) Calculation of range.

Range = Highest Score Lowest Score

= 15 - 4 = 11

(2) Calculation of quartile deviation

$$Q_1 = L + \frac{\frac{N}{4} - F}{f} \times CI$$

$$= 6.5 + \frac{\frac{12}{4} - 2}{1} \times 1 = 6.5 + 1 = 7.5$$

$$Q_3 = L + \frac{\frac{3N}{4} - F}{f} \times CI$$

$$= 12.5 + \frac{\frac{3 \times 12}{4} - 8}{2} \times 1 = 12.5 + 0.5 = 13$$

$$QD = \frac{Q_3 - Q_1}{2} = \frac{13 - 7.5}{2} = \frac{5.5}{2} = 2.75$$

X	f	F
15	2	12
13	2	10
11	1	8
10	3	7
8	1	4
7	1	3
4	2	2
Total	12	

(3) Calculation of mean deviation

$$\overline{X} = \frac{\sum X}{N} = \frac{120}{12} = 10$$

$$MD = \frac{\sum |X - \overline{X}|}{N} = \frac{34}{12} = 2.83$$

X	$X - \overline{X}$	$\lvert X - \overline{X} \rvert$	$(X - \overline{X})^2$
10	0	0	0
15	5	5	25
13	3	3	9
8	-2	2	4
10	0	0	0
10	0	0	0
15	5	5	25
13	3	3	9
11	1	1	1

(4) Calculation of standard deviation

$$SD = \sqrt{\frac{\sum (X - X)^2}{N}}$$

$$= \sqrt{\frac{154}{12}}$$

$$= \sqrt{12.83}$$

$$= 3.58$$

7	-3	3	9
4	-6	6	36
4	-6	6	36
Total		34	154

(5) Calculation of variance

$$\text{Variance} = \frac{\sum(X-\overline{X})^2}{N} = \frac{154}{12} = 12.83$$

Note : (1) Variance = SD^2 (2) SD = Square root of Variance

Calculation of Measures of Variability from Grouped Data

Example (2) Find range, quartile deviation, standard deviation and variance of the following frequency distribution table.

X	f
70-74	5
65-69	5
60-64	8
55-59	10
50-54	20
45-49	14
40-44	12
35-39	16
30-34	10
Total	100

For calculating such measures, we have to construct the given table adding with another four columns. (F, *d, fd, fd*^{2}) as follows:

X	*f*	F	*d*	*fd*	*fd*^{2}
70-74	5	100	4	20	80
65-69	5	95	3	15	45
60-64	8	90	2	16	32
55-59	10	82	1	10	10
50-54	20	72	0	0	0
45-49	14	52	-1	-14	14
40-44	12	38	-2	-24	48
35-39	16	26	-3	-48	144
30-34	10	10	-4	-40	160
Total	100				533

(1) Calculation of range

Range = Highest Score – Lowest Score = 72 – 32 = 40

(2) Calculation of quartile deviation

$$Q_1 = L + \frac{\frac{N}{4} - F}{f} \times CI = 34.5 + \frac{25-10}{16} \times 5 = 39.18$$

$$Q_3 = L + \frac{\frac{3N}{4} - F}{f} \times CI = 54.5 + \frac{75-72}{10} \times 5 = 56$$

$$Q = \frac{Q_3 - Q_1}{2} = \frac{56 - 39.18}{2} = 8.41$$

(3) Calculation of Standard Deviation

$$SD = CI \times \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} = 5 \times \sqrt{\frac{533}{100} - \left(\frac{-65}{100}\right)^2}$$

$= 5 \times 2.21 \qquad = 11.05$

(4) Calculation of Variance

Variance $= (SD)^2 \qquad = (11.05)^2 \qquad = 122.1025$

Exercises

1. The data below represent raw scores on a test taken by a class of 15 students.

32	36	38	39	42
44	45	46	45	45
48	48	50	51	51

(a) Find the range of distribution.

(b) Compute mean deviation and standard deviation by using the ungrouped data method.

(c) Arrange the scores into frequency distribution with intervals three points wide (i.e. CI = 3) beginning with 30-32. Compute again the standard deviation and quartile deviation of the given distribution.

2. If $\sum\left(X - \overline{X}\right)^2 = 144$, N = 16, find the variance and standard deviation.

Percentile and Percentile Rank

The Nature of Percentile and Percentile Rank

A numerical value which summarizes the responses actually made on a test by an individual is called a raw score. Raw score alone is not enough to interpret a student's performance. Knowing a student's raw score merely is about how well he did on the test and it also means that how much a student about how well he did on the test and it also means that how much a student made his/her actual performance without comparing to either a criterion score or the performance of others who took the same test. Clearly there is a need for methods of transforming raw scores into values which facilitate the interpretation of scores on both an individual and group basis. Then it becomes the measures of relative position more meaningful in interpretation of scores and in comparison of student's performance as well.

Measures of relative position indicate where a score is in relation to all other scores in the distribution. In other words, measures of relative position permit a teacher to express how well an individual has performed as compared to all other individuals in same group who have been given the same test. Raw scores that have been transformed systematically into equivalent values which indicate relative position are referred to as derived scores. The most common types of derived scores are percentile and percentile ranks.

A percentile is a point which cuts off a given percentage of a distribution.

There are 99 percentiles that a distribution is divided into 100 equal parts. The first percentile symbolized as P_1 is the point below which are one percent of the scores. Similarly, P_{50} is the point below which 50% of the scores and P_{90} is the point below which are 90% of the scores.

A percentile rank (PR) indicates the percentage of scores that lie below a given value in the total distribution.

Thus, if a score of 48 corresponds to a percentile rank of 80 (symbolized PR80), this means that 80% of the score are below the score 48. In other words, if a student had a percentile rank of 98, this would mean that he/she did as well or better than 98% of the members of some group which took the same test.

The median of a set of scores corresponds to the 50^{th} percentile which makes sense since the median is the middle point and therefore the point below which are 50% of the scores. Similarly, the first quartile (Q1) corresponds to the 25^{th} percentile and second quartile (Q2) corresponds to the 50^{th} percentile and the third quartile (Q3) corresponds to the 75^{th} percentile. thus, percentile ranks basically allow us to determine how well an individual did in relative terms, as compared to others who took the same test. If percentile ranks are given for a number of subtests, they also provide a rough mean if comparing an individual's relative performance in a number of different areas.

One point to keep in mind when interpreting percentile ranks is that they are ordinal, not interval measures. Therefore, we do not have equal intervals between percentile points. An increase of a given number of percentile points corresponds to a different number of ranks score points depending upon where they are in the distribution. For example, the difference between the 45^{th} and 50^{th} percentile does not represent the same increase in raw scores as the difference between the 90^{th} and 95^{th} percentiles.

Uses of Percentile and Percentile Rank

The percentile rank is useful to compare not only the performance of two different tests. It can be used to set up tables that list the percentile equivalents of each raw score. Moreover, it can also to draw a profile of the student's performance (relative to a reference group) on a several different tasks.

Percentiles are also useful in determining the cutoff score. Cutoff scores are never predetermined in a relative system. Percentile ranks must be set up first according to the quota. After that percentiles must be computed to find the raw scores equivalent to the percentile ranks which have already been set up. For example, 20% of the candidates who sit for a competition examination is supposed to be selected. It means that 80% will be rejected. Thus, P_{80} must be computed to determine a raw score, below which 80% of total candidates lie. This calculated raw scores can be regarded as the cutoff score, because students who receive below this raw score will be rejected. It is obvious that percentile and percentile rank are interdependent.

The Computation of Percentiles and Percentile Ranks

The method of calculating percentiles is essentially the same as that employed in finding the median and quartiles. So, it can be calculated by using the following formula.

$$P_p = L + \frac{\frac{\text{P x N}}{100} - \text{F}}{\text{f}} \text{ x } CI$$

Where, P = Percentage of the distribution wanted.

L = exact lower limit of class interval upon which P_p lies.

$\frac{\text{P x N}}{100}$ = Part of N to be counted off in order to reach P_p.

F = Sum of all scores upon intervals below L. This is, the cumulative frequency below the class containing P_p.

CI = Length of the class interval.

N = number of cases.

Calculation of Percentiles from Ungrouped Data

Example (1)

Find P_{70} of English test scores of ten students given below.

70, 60, 55, 55, 50, 46, 46, 46, 40, 35

In order to make the computation easier, it is necessary to set up a frequency distribution table using the given data shown in the table.

X	f	F
70	1	10
60	1	9
55	2	8
50	1	6
46	3	5
40	1	2
35	1	1
Total	10	

The formula for P_{70} can be written as:

$$P_{70} = L + \frac{\frac{70N}{100} - F}{f} \text{ x } CI$$

$$\frac{70N}{100} = \frac{70 \times 10}{100} = 7$$

Therefore from the table, it can be observed that class interval in which P_{70} falls is 5.5.

$$\therefore L = 54.5,\ F = 6,\ f = 2,\ CI = 1$$

$$\therefore P_{70} = 54.5 + \frac{7-6}{2} \text{ x } 1$$

$$\therefore P_{70} = 54.5 + 0.5$$
$$\therefore P_{70} = 55$$

Calculation of Percentile from Grouped Data

Example (2)

Find P_{70} by using the following frequency distribution table. The table is showing the Mathematics test scores of 100 students of Standard Five.

X	f
70-74	5
65-69	8
60-64	7
55-59	10
50-54	30
45-49	15
40-44	18
35-39	4
30-34	3
Total	100

Before applying the formula, it is necessary to reconstruct the table with its cumulative frequencies as follows.

X	f	F
70-74	5	100
65-69	8	95
60-64	7	87
55-59	10	80
50-54	30	70
45-49	15	40
40-44	18	25
35-39	4	7
30-34	3	3
Total	100	

$$P_{70} = L + \frac{\frac{70\,N - F}{100}}{f} \text{ x } CI$$

$$\frac{70N}{100} = \frac{70 \text{ x } 100}{100} = 70$$

$$\therefore L = 49.5, , F = 40, f = 30, CI = 5$$

$$\therefore P_{70} = 49.5 + \frac{70 - 40}{30} \text{ x } 5$$

$$\therefore P_{70} = 49.5 + 5$$

$$\therefore P_{70} = 54.5$$

Calculation of Percentiles Ranks from Ungrouped Data

The formula for calculating percentile rank is

$$PR = \frac{\text{No. of cases below the score + .5(No. of cases at the score)}}{N} \text{ x } 100$$

Example (3) Compute PR of the student with the test score of 50 in the distribution, 30, 32, 33, 38, 40, 48, 50, 58, 70, 75

By the problem,

Number of cases below the score 50 = 6

Number of cases at the score 50 = 1

Total number of students in the distribution = 10

$$PR = \frac{6 + 0.5 \times 1}{10} \text{ x } 100$$

$$= \frac{6.5}{10} \text{ x } 100$$

$$= 65\%$$

Therefore, percentile rank of score 50 is 65%

The Computation of Percentile rank from grouped data

If percentile ranks have to be computed from a frequency distribution, where the scores are grouped within the range of 2 intervals, we have to use the following formula.

$$PR = \left[(S - L)\frac{f}{CI} + F \right]\frac{100}{N}$$

where PR = percentile rank

S = the given score

L = the lower limit of the class interval in which the given score lies

f = frequency of the class interval in which the given score lies

F = cumulative frequency below the class containing the given score

CI = class interval

Example (4) Find PR of the score 67 in the following frequency distribution.

X	f
70-74	5
65-69	8
60-64	7
55-59	10
50-84	30
45-49	15
40-44	18
35-39	4
30-34	3
Total	100

The given table is reconstructed as follows and *L, f, F* and *CI* are calculated from that table.

X	*f*	F
70-74	5	100
65-69	8	95
60-64	7	87
55-59	10	80
50-54	30	70
45-49	15	40
40-44	18	25
35-39	4	7
30-34	3	3
Total	100	

Score 67 falls on the interval (65-69)

L = 64.5, f = 8, F = 87, CI = 5, S = 67, N = 100

$$PR = \left[(S-L)\frac{f}{CI}+F\right]\frac{100}{N}$$

$$= \left[(67-64.5)\frac{8}{5}+87\right]\frac{100}{100}$$

$$= \left[2.5\times\frac{8}{5}+87\right]\times 1$$

$$= [4 + 87] \text{ x } 1 = 91\%$$

Therefore, percentile rank of score 67 is 91%.

Exercise

(1) By using the data given in example (4), find P_{91}.

(2) By using the data given in example (4), find how many percentages of students who passed the examination when their

cutoff score is decided as 40 marks.

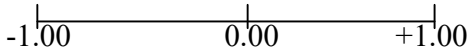
(3) By observing the given data also, find cutoff score so that 90% of students should pass the examination.

Correlation

So far we have been concerned only with a single variable and its frequency distribution. Now in this chapter, our concern will be on two variables or even more.

Many variables or events in nature are related to each other. As the sun rises, the day warms up, as children becomes matured, they think more completely and the persons bright in one area tend to be bright in certain areas too. Such relationships are called correlations. This relationship of one variable to another is known as correlation.

The correlation coefficient is a statistical measure that describes the degree of relationship between two or more variables. All correlation coefficients lie between – 1.00 and +1.00 passing through 0.00. One might more easily visualize possible coefficients as existing on a line as indicated below.



The sign of coefficient expresses the direction of the relationship, and the size of coefficient expresses the degree of relationship.

A scatter diagram is one of the clearest ways of showing the meaning of the correlation graphically. By using scatter diagrams the student can get the indication of the magnitudes of the correlation coefficient and also an indication of the signs of relationship.

Classification of Correlation

Correlation can be classified into five categories as follows:

(1) Perfect positive correlation.

(2) Positive correlation

(3) Perfect negative correlation

(4) Negative correlation

(5) Zero correlation

(1) Perfect positive correlation

As X increases a given amount, Y increases in a proportionate amount. The relationship is therefore a perfect positive one.

X: 20, 18, 16, 14, 12, 10, 8, 6, 4, 2

Y: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1& was To

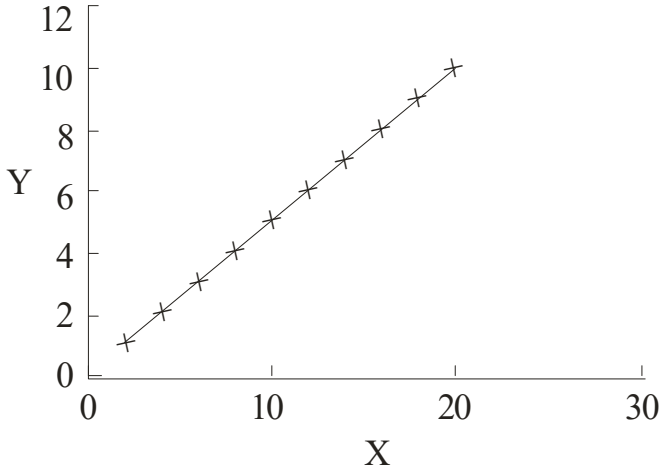


Fig (1)

When the pairs of these variables are plotted, the points will fall along a straight line. It runs upward from left to right as shown in Fig (1). The correlation coefficient r is equal to 1.00.

(2) Positive correlation

But if all pairs of variables do not progress together unit for unit, their relationship is positive but less than perfect.

X (Age): 43, 48, 56, 61, 67, 70

Y (Pressure): 128, 120, 135, 143, 141, 152

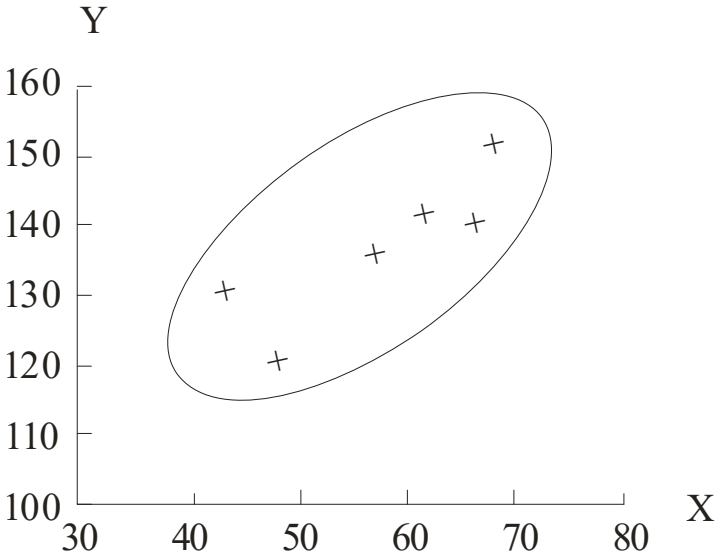


Fig (2)

Thus the plotted points fall near but not directly on the straight line as shown in Fig (2). The correlation coefficient would be less than 1.00.

Some example of positively correlated variables are intelligence and school success, high

rainfall and high humidity.

(3) Perfect Negative Correlation

For every increase on the X, Y decreases a constant number of units, the relationship therefore is a perfect negative one. When the pairs of the variables are plotted, the points will fall along a straight line. It runs downward from left to right as shown in Fig (3). The correlation coefficient r equals to – 1.00.

X: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

Y: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1

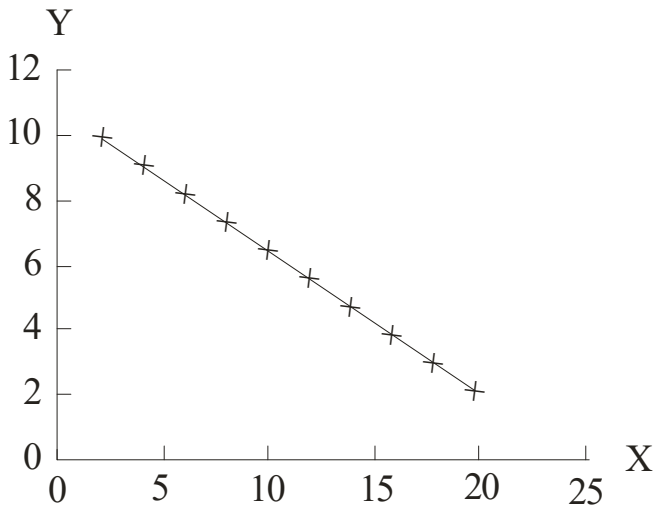


Fig (3)

(4) Negative Correlation

As X increases a unit, Y has a decreasing trend but not in a proportionate amount. The relationship therefore is negative but not perfect.

When the pairs of the variables are plotted, it will not fall along a straight line, as shown in Fig (4). The correlation coefficient r will be a negative number between 0.00 and – 1.00.

Some examples of negative correlation are fatigue and test performance, decrease in teaching quality and increase in number of failures.

X (No. of Absence) 6, 2, 15, 9, 12, 5, 8

Y (Final Grade) 82, 86, 43, 74, 58, 90, 78

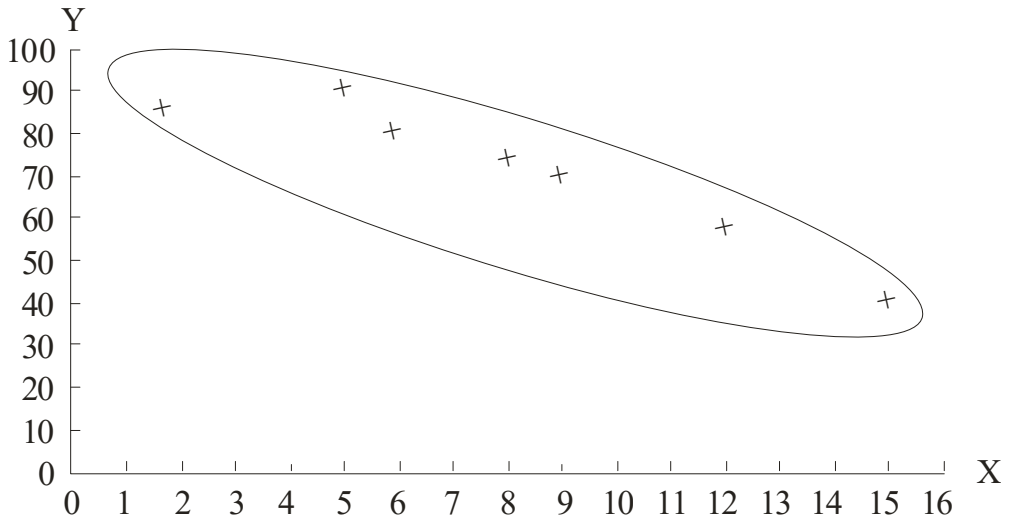


Fig (4)

(5) Zero Correlation

If an increase or decrease in one variable tells us nothing about the likely condition of the other variable, there is no relationship, and we may say that there is zero correlation.

When the pairs of the variables are plotted, we will have no definite direction as shown in Fig (5). The correlation r will be zero.

An example of zero correlation is adult height and intelligence.

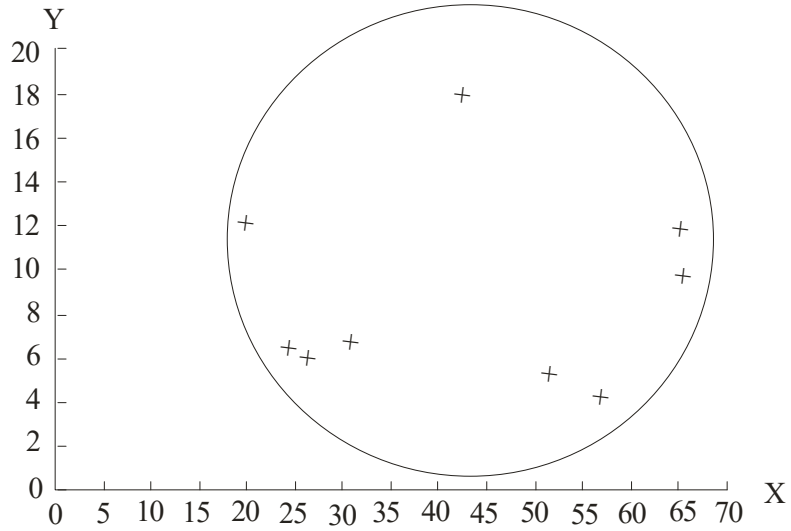


Fig (5)

The Strength of Correlation Coefficient

When statisticians refer to high and low correlation, they are usually using the absolute scale of 1.00 to 0.00 as their guide. 'High' or 'Strong' correlation coefficients therefore, should not necessarily be thought as 'important' nor should 'low' or 'weak' correlation coefficients necessarily be considered 'unimportant'.

Generally correlation coefficient r equal to or greater than 0.70 can be regarded as high,

between 0.40 and 0.70 as fair and equal to or less than 0.40 as low correlation.

But it should be noted that for certain purposes r of 0.50 might be considered satisfactory whereas in other situations, an r of 0.90 or high would be required.

If two test forms are to be considered equivalent, we would expect a very high positive correlative between scores on the two forms, i.e. a correlation of somewhat about 90. Yet in other educational situations, for example, in relating academic achievement to a predictor test of achievement, such as IQ test, we are often satisfied with an r of between 0.40 and 0.50. An r of 0.70 in such situation would be exceptional indeed.

The Computation of the Pearson Product Moment Correlation Coefficient

Computing a Correlation Coefficient from Raw Scores

Formula:

$$r = \frac{\sum XY - \frac{\sum X \sum Y}{N}}{\sqrt{\left[\sum X^2 - \frac{(\sum X)^2}{N}\right]\left[\sum Y^2 - \frac{(\sum Y)^2}{N}\right]}}$$

Examples

(1) Given the following scores on an arithmetic test and corresponding scores on a reading comprehension test, correlate the two set of scores.

X Arithmetic	Y Reading	X^2	Y^2	XY
3	6	9	36	18
2	4	4	16	8
4	4	16	16	16
6	7	36	49	42
5	5	25	25	25
1	3	1	9	3
$\sum X = 21$	$\sum Y = 29$	$\sum X^2 = 91$	$\sum Y^2 = 151$	$\sum XY = 112$

$$r = \frac{112 - \frac{21 \times 29}{6}}{\sqrt{\left[91 - \frac{(21)^2}{6}\right]\left[151 - \frac{(29)^2}{6}\right]}}$$

$$= \frac{112 - \frac{609}{6}}{\sqrt{\left[91 - \frac{441}{6}\right]\left[151 - \frac{841}{6}\right]}}$$

$$= \frac{112 - 101.5}{\sqrt{[91 - 73.5][151 - 140.16]}}$$

$$= \frac{10.5}{\sqrt{17.5 \times 10.84}}$$

$$= \frac{10.5}{\sqrt{189.7}} \quad = \frac{10.5}{13.77} \quad = 0.76$$

Computer r using the following information.

$\sum X^2 = 8.5$ $\sum X = 83$ $\sum XY = 788$

$\sum Y^2 = 766$ $\sum Y = 82$ $N = 10$

$$r = \frac{\sum XY - \frac{\sum X \sum Y}{N}}{\sqrt{\left[\sum X^2 - \frac{(\sum X)^2}{N}\right]\left[\sum Y^2 - \frac{(\sum Y)^2}{N}\right]}}$$

$$= \frac{788 - \frac{83 \times 82}{10}}{\sqrt{\left[835 - \frac{(83)^2}{10}\right]\left[766 - \frac{(82)^2}{10}\right]}}$$

$$= \frac{788 - 680.6}{\sqrt{[835 - 688.9][766 - 672.4]}}$$

$$= \frac{107.4}{\sqrt{146.1 \times 688.9}}$$

$$= \frac{107.4}{\sqrt{13674.96}}$$

$$= \frac{107.4}{116.94}$$

$$= 0.918$$

$$= 0.92$$

Computing a Correlation Coefficient from Standard Sores or z Scores.

Formula: $r_{xy} = \frac{\sum Z_x Y_y}{N}$

where $Z_X = \frac{X - \overline{X}}{\sigma_x}$, $Z_Y = \frac{Y - \overline{Y}}{\sigma_y}$

N = number of pairs of X, Y scores.

Example

Given the following pairs of z scores rep children's position on the IQ scale an on a reading reading comprehension?

IQ scale = x

Reading Comprehension = y

Z_x	Z_y	Z_xZ_y
1.4	1.0	1.40
1.0	1.4	1.40
.7	.7	.49
.0	.0	.00
-.6	-1.2	.72
-1.3	-.6	.78
-1.2	-1.3	1.56
		$\sum Z_xZ_y$=6.35

$$r_{xy} = \frac{\sum Z_x Z_y}{N}$$

$$= \frac{6.35}{7}$$

$$= 0.91$$

Exercises

(1) Ten individuals made the following scores on two methods (X and Y) of spelling test. Compute r between their scores.

X	Y
14	13
12	15
10	8
9	10
8	7
7	5
6	4
3	2
2	4
2	5

(2) By using the following information, calculate r.

$\sum X^2 = 393$ $\sum X = 57$ $\sum XY = 431$

$\sum Y^2 = 514$ $\sum Y = 62$ $N = 10$

(3) Calculate r from the given pairs of Z scores.

Z_x	Z_y
1.50	0.50
2.00	1.00
0.90	-0.30
-0.35	2.00
-3.00	-0.25
0.50	0.10
0.30	0.20

EXERCISE

1. What are the limitations of objective type test items?
2. List the steps involved in construction of an achievement test
3. Explain item analysis
4. What do you mean by diagnosis?
5. List the essential for the construction of a diagnostic test
6. Explain the concept of diagnostic testing. How is it different from achievement testing? Illustrate the difference with the help of an example.
7. Describe the role of diagnostic testing in continuous and comprehensive evaluation.
8. What if Remedial teaching? Describe various techniques/measures, which can be used in remedial teaching.
9. Distinguish between the achievement test and diagnostic test in their functions and preparation.
10. What is meant by a diagnostic test? How do you prepare it?
11. What are the advantages of objective type test items?
12. Explain what is diagnosis?
13. List the levels of diagnosis
14. Describe the characteristics of diagnosis
15. Explain what do you mean by remediation?
16. What do you mean by educational diagnosis?
17. List the levels of diagnosis.
18. What is Norm referenced tests?
19. Give an account of criterion referenced test.
20. Compare Norm referenced and criterion referenced tests.
21. How will you prepare a frequency distribution table?
22. What are the main uses of a frequency distribution in statistical analysis?